Name: Key

Math 123: Trigonometry
Midterm Exam # 4
26 November 2013

Follow all instructions. All scratch work should be done on the paper provided. You may use a calculator, but no other electronic devices.

Part I: True or False

Read each statement carefully, then write **T** *or* **F** *in the space provided.*

F **1.** $3 + 4i = 5(\cos(\pi/4) + i\sin(\pi/4))$ $\theta \neq \pi/4$

T **2.** For every nonzero complex number $z = a + bi$, there are unique real numbers $r > 0$ and $0 \leq \theta < 2\pi$ such that $z = r(\cos\theta + i\sin\theta)$.

T **3.** Every complex number $z = x + iy$ can be identified with a vector $\langle x, y \rangle$.

F **4.** It is possible for a third-degree polynomial with integer coefficients to have no real zeros. complex roots come in pairs, so the third must be real.

T **5.** If p is a polynomial of degree n, $n > 0$, then p has at least one zero in the complex number system. The Fundamental Theorem of Algebra!

Part II: Fill in the Blank

Choose the appropriate answer from the word bank, and write its corresponding letter in the space provided.

Word Bank:

A. conjugates	**B.** $-i$	**C.** imaginary
D. pairs	**E.** discriminant	**F.** root(s)
G. determinant	**H.** real	**I.** -1
J. i	**K.** quadratic formula	**L.** 1

H **6.** A(n) ______ number has the form $a + bi$, where $a \neq 0$ and $b = 0$.

A **7.** Two complex numbers $a + bi$ and $a - bi$ are called complex ______ .

J **8.** $i^{1997} =$______ .

B **9.** $\dfrac{1}{i} =$______ .

E **10.** The expression $b^2 - 4ac$ is called the ______ of the quadratic equation $ax^2 + bx + c = 0$.

Part III: Multiple Choice

Write the letter corresponding to the appropriate answer in the space provided.

D **11.** Determine the nature of the roots of the equation: $2x^2 - 5x + 5 = 0$.

A. one real solution **B.** two distinct real solutions

C. one complex solution **D.** two complex solutions

C **12.** Perform the operation: $\dfrac{3+i}{-3+4i}$.

A. $-1-3i$ **B.** $-5-15i$

C. $\dfrac{-1}{5} - \dfrac{3}{5}i$ **D.** $\dfrac{-1}{25} - \dfrac{3}{25}i$

D **13.** Find all roots of the equation: $2x^2 - 8x = -10$

A. $2, \ -2$ **B.** $2i, \ -2i$

C. $1+2i, \ 1-2i$ **D.** $2+i, \ 2-i$

For problems **14** and **15**, consider the complex number $z = -4 + 4i$.

C **14.** Find the modulus $|z|$.

A. 32 **B.** $8\sqrt{2}$

C. $4\sqrt{2}$ **D.** 16

A **15.** Find the argument θ.

A. $\dfrac{3\pi}{4}$ **B.** $\dfrac{\pi}{4}$

C. $\dfrac{-\pi}{4}$ **D.** $\dfrac{-3\pi}{4}$

C **16.** Find a polynomial p such that 2 and -3 are zeros, and $p(0) = 3$.

A. $p(x) = (x+2)(x-3)$ **B.** $p(x) = 2(x-2)(x+3)$

C. $p(x) = -\frac{1}{2}(x-2)(x+3)$ **D.** $p(x) = \frac{1}{2}(x-2)(x+3)$

A **17.** How many times does the graph of $y = 2x^2 - 8x + 9$ cross or touch the x-axis?

A. 0 **B.** 1

C. 2 **D.** I don't know, dude.

D **18.** A ball is kicked upward from ground level with an initial velocity of 48 ft/sec. The height h (in ft) is given by $h(t) = -16t^2 + 48t$, $0 \le t \le 3$, where t is the time (in sec). After reaching its peak and beginning to fall back toward the ground, at what time is the ball at a height of 32 ft?

A. 1 sec **B.** 2.5 sec

C. 1.5 sec **D.** 2 sec

Part IV: Short Answer

Show enough work. Clearly mark your final answers. Partial credit given when deserved.

19. Find a polynomial function of lowest degree that has $3i$ and 2 as zeros.

Zeros: $3i, -3i, 2$

$p(x) = (x+3i)(x-3i)(x-2)$

$= (x^2+9)(x-2)$

$\boxed{p(x) = x^3 - 2x^2 + 9x - 18}$

20. Let $z_1 = 18(\cos(60°) + i\sin(60°))$ and $z_2 = 6(\cos(30°) - i\sin(30°))$. Find $z_1 z_2$ and z_1/z_2. Give your answers in standard form $(a + bi)$.

$z_2 = 6(\cos(-30°) + i\sin(-30°))$

$z_1 z_2 = 108(\cos(30°) + i\sin(30°)) = 108\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = \boxed{54\sqrt{3} + 54i}$

$\frac{z_1}{z_2} = \frac{18}{6}(\cos(90°) + i\sin(90°)) = 3(0 + 1i) = \boxed{3i}$

21. Given that $3 - i$ is a zero of the polynomial $f(x) = x^3 - 2x^2 - 14x + 40$, find all other zeros. Write f as a product of linear factors.

If $3-i$ is a zero, so is $3+i$

Then $(x-(3-i))(x-(3+i))$ is a factor.

$= x^2 - (3-\cancel{i})x - (3+\cancel{i})x + 9 + 1$

$= x^2 - 6x + 10$

divide:

$$\begin{array}{r|l} & x + 4 \\ x^2 - 6x + 10 & x^3 - 2x^2 - 14x + 40 \\ - & x^3 + 6x^2 + 10x \\ \hline & 4x^2 - 24x + 40 \\ & 4x^2 - 24x + 40 \\ \hline & 0 \end{array}$$

zeros: $3-i, 3+i, -4$

$f(x) = (x - 3 + i)(x - 3 - i)(x + 4)$