Name: Key

Math 123: Trigonometry
Midterm Exam # 3
7 November 2013

Follow all instructions. All scratch work should be done on the paper provided. You may use a calculator, but no other electronic devices.

Part I: True or False
Read each statement carefully, then write **T** *or* **F** *in the space provided.*

F 1. The equation $\sin(x) = 1$ has only one solution. $x = \frac{\pi}{2} + 2\pi n$

F 2. For all vectors $\vec{\mathbf{v}} = \langle v_1, v_2 \rangle$, $\vec{\mathbf{v}} \cdot \vec{\mathbf{0}} = \vec{\mathbf{0}}$. $\vec{v} \cdot \vec{0} = 0$, not $\vec{0}$

F 3. Two vectors are equivalent if they have the same direction angle. magnitude also.

T 4. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$; then the vector $\vec{PQ}$ is given in component form by $\langle x_2 - x_1, y_2 - y_1 \rangle$.

F 5. Consider the vector $\vec{\mathbf{v}} = \langle x, y \rangle$ with $x > 0$ and $y \geq 0$. Its direction angle is $\theta = \cos^{-1}\left(\frac{y}{x}\right)$. $\theta = \tan^{-1}(y/x)$

Part II: Fill in the Blank
Choose the appropriate word or phrase from the word bank, and write its corresponding letter in the space provided.

Word Bank:

A. Ambiguous	**B.** Law of Averages	**C.** Law of Cosines
D. Obvious	**E.** Heron's Formula	**F.** Wrong
G. Pythagorus's Formula	**H.** Factorization	**I.** Law of Sines
J. Cross Product	**K.** Oblique	**L.** Dot Product

K 6. A(n) ______ triangle has no right angle.

C 7. $c^2 = a^2 + b^2 - 2ab\cos(C)$.

E 8. Area $= \sqrt{s(s-a)(s-b)(s-c)}$.

A 9. "Angle–Side–Side" is known as the ______ case.

L 10. The ______ can be used to find the angle between two vectors.

Part III: Multiple Choice

Write the letter corresponding to the appropriate answer in the space provided.

__C__ **11.** Solve: $\sin^2(x) - \sin(x) + \frac{1}{4} = 0$ in $[0, 2\pi)$.

A. π; 2π **B.** $\frac{\pi}{4}$; $\frac{\pi}{2}$

C. $\frac{\pi}{6}$; $\frac{5\pi}{6}$ **D.** $\frac{\pi}{3}$; $\frac{2\pi}{3}$

__D__ **12.** Solve: $\tan(\frac{x}{4}) = -1$.

A. $\pi + 4\pi n$ **B.** $\frac{7\pi}{4} + \pi n$

C. $-\frac{\pi}{4} + \pi n$ **D.** $7\pi + 4\pi n$

For problems 13 through 15: Consider the oblique triangle with $a = 4$, $A = \frac{\pi}{4}$, and $B = \frac{\pi}{3}$.

__D__ **13.** Find b.

A. $6\sqrt{2}$ **B.** 5

C. 3 **D.** $2\sqrt{6}$

__A__ **14.** Find C.

A. $\frac{5\pi}{12}$ **B.** $\frac{7\pi}{12}$

C. $\frac{\pi}{2}$ **D.** $\frac{\pi}{6}$

__B__ **15.** Find the area of the triangle ΔABC.

A. 8.27 **B.** 9.46

C. 18.92 **D.** 9.80

C 16. Let $\vec{\mathbf{u}} = \langle 3, -7 \rangle$ and $\vec{\mathbf{v}} = \langle 12, 5 \rangle$. Find $\text{proj}_{\vec{\mathbf{v}}}\vec{\mathbf{u}}$.

A. $\left\langle \frac{3}{12}, \frac{-7}{5} \right\rangle$ **B.** $\left\langle \frac{12}{13}, \frac{5}{13} \right\rangle$

C. $\left\langle \frac{12}{169}, \frac{5}{169} \right\rangle$ **D.** $\langle 3, 5 \rangle$

C 17. How many possible triangles are there with $a = 6$, $b = 6.5$, and $A = 58°$?

A. 0 **B.** 1

C. 2 **D.** I don't know, dude.

B 18. Find a unit vector orthogonal to the vector $\vec{\mathbf{v}} = \langle -4, -3 \rangle$.

A. $\left\langle -\frac{4}{5}, -\frac{3}{5} \right\rangle$ **B.** $\left\langle \frac{3}{5}, -\frac{4}{5} \right\rangle$

C. $\langle -3, 4 \rangle$ **D.** $\left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$

Part IV: Short Answer

Show enough work. Clearly mark your final answers. Partial credit given when deserved.

19. Let $\vec{\mathbf{u}} = \langle -3, 2 \rangle$ and $\vec{\mathbf{v}} = \langle 1, 1 \rangle$. Find $\vec{\mathbf{w}}_1 = \text{proj}_{\vec{\mathbf{v}}}\vec{\mathbf{u}}$ and $\vec{\mathbf{w}}_2 = \text{orth}_{\vec{\mathbf{v}}}\vec{\mathbf{u}}$.

$\vec{w}_1 = \text{proj}_{\vec{v}}\vec{u} = \left(\frac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v} = \left(\frac{-1}{2}\right)\langle 1, 1 \rangle = \boxed{\langle -\tfrac{1}{2}, -\tfrac{1}{2} \rangle}$

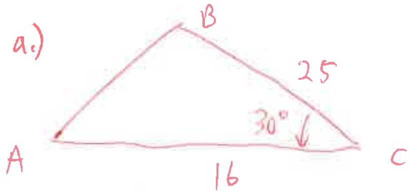
$\vec{u}\cdot\vec{v} = -3+2 = -1$

$\vec{v}\cdot\vec{v} = 1+1 = 2$

$\vec{w}_2 = \text{orth}_{\vec{v}}\vec{u} = \vec{u} - \vec{w}_1 = \langle -3, 2 \rangle - \langle -\tfrac{1}{2}, -\tfrac{1}{2} \rangle = \boxed{\langle -\tfrac{5}{2}, \tfrac{5}{2} \rangle}$

20. (*a.*) Solve for the triangle with $a = 25$, $b = 16$, and $C = 30°$. Round all answers to one decimal place, if necessary.
(*b.*) Find the area of the triangle. Give the exact answer.

a.)



$$c^2 = 16^2 + 25^2 - 2(16)(25)\cos(30°)$$

$$= 256 + 625 - 800\left(\tfrac{\sqrt{3}}{2}\right) = 881 - 800\left(\tfrac{\sqrt{3}}{2}\right)$$

$$\approx 188.18$$

so $c \approx \sqrt{188.18} \approx \boxed{13.7 = c}$

$$\sin B = \frac{b \sin C}{c} = \frac{16 \sin(30°)}{13.7} = \frac{16(1/2)}{13.7} = \frac{8}{13.7} = 0.5839$$

So $B = \sin^{-1}(0.5839) \approx \boxed{35.7° = B}$

Then $A = 180° - B - C = 180° - 35.7° - 30° = 180° - 65.7° = \boxed{114.3° = A}$

b.) $\text{Area} = \frac{1}{2}ab\sin(C) = \frac{1}{2}(16)(25)\sin(30°) = 8(25)(1/2) = 4(25) = \boxed{100 = \text{Area}}$

21. Prove the Cauchy-Schwarz Inequality: $|\vec{\mathbf{u}} \cdot \vec{\mathbf{v}}| \leq \|\vec{\mathbf{u}}\| \, \|\vec{\mathbf{v}}\|$.

Recall that $\vec{u} \cdot \vec{v} = \|\vec{u}\| \, \|\vec{v}\| \cos\theta$, so

$$|\vec{u} \cdot \vec{v}| = \big|\|\vec{u}\| \, \|\vec{v}\| \cos\theta\big|$$

$$= \|\vec{u}\| \, \|\vec{v}\| \, |\cos\theta|$$

but $|\cos\theta| \leq 1$, so we get

$$\boxed{|\vec{u} \cdot \vec{v}| \leq \|\vec{u}\| \, \|\vec{v}\|} \quad \square$$