Name: KEY

Math 123: Trigonometry
Midterm Exam # 2
Due: 8 October 2013

Follow all instructions. Late submissions will not be accepted.

Part I: True or False [2 points each]
Read each statement carefully, then write **T** *or* **F** *in the space provided.*

F **1.** $\arcsin(\sin(x)) = x$ for all real values of x. range arcsin is $[-\pi/2, \pi/2]$

T **2.** $\tan(\arctan(x)) = x$ for all real values of x.

F **3.** The range of every arc-function is the right-hand side of the standard unit circle. No, Cosine is the top half.

T **4.** $\sin^2(x)\cot(x) = \frac{\cos(x)}{\csc(x)}$.

T **5.** $\csc^{-1}(2) = \frac{\pi}{6}$.

Part II: Fill in the Blank [2 points each] *Choose the appropriate word or phrase from the word bank, and write its corresponding letter in the space provided.*

Word Bank:

A. quadratic	**B.** $(-1, 1)$	**C.** $(-\infty, \infty)$
D. Pythagorean	**E.** power-reducing	**F.** $[-1, 1]$
G. $[-\frac{\pi}{2}, \frac{\pi}{2}]$	**H.** double-angle	**I.** half-angle
J. $[0, \pi]$	**K.** $(-\frac{\pi}{2}, \frac{\pi}{2})$	**L.** sum-to-product

C **6.** The domain of $\arctan(x)$ is _____ .

J **7.** The range of $\arccos(x)$ is _____ .

D **8.** $\sin^2(x) + \cos^2(x) = 1$ is called a _____ identity.

H **9.** $\sin(2u) = 2\sin(u)\cos(u)$ is an example of a _____ formula.

F **10.** The domain of $\arcsin(x)$ is _____ .

Part III: Multiple Choice [5 points each]
Write the letter corresponding to the appropriate answer in the space provided.

C **11.** Rewrite the algebraic expression as a trig expression: $\sqrt{9-x^2}$; $x = \cos(\theta)$.

A. **B.**

C. $3\sin(\theta)$ **D.**

A **12.** Write an equivalent expression for $\sec(\theta)$ in terms of $\sin(\theta)$; $0 < \theta < \frac{\pi}{2}$.

A. $\frac{1}{\sqrt{1-\sin^2(\theta)}}$ **B.**

C. **D.**

B **13.** Use the cofunction identity to evaluate the expression without a calculator: $\sin^2(71°) + \sin^2(19°)$.

A. **B.** 1

C. **D.**

B **14.** Simplify: $\dfrac{\sec^2(\theta) - 1}{\sec^2(\theta)}$

A. **B.** $\sin^2(\theta)$

C. **D.**

D **15.** Simplify: $\sin\big(\arcsin(x) + \arccos(x)\big)$; $-1 \leq x \leq 1$.

A. **B.**

C. **D.** 1

A **16.** Find the exact value of $\sin(\frac{11\pi}{12})$. [Hint: $\frac{11}{12} = \frac{3}{4} + \frac{1}{6}$.]

A. $\frac{\sqrt{3}-1}{2\sqrt{2}}$ **B.**

C. **D.**

C **17.** Evaluate: $\arcsin(\sin(\frac{15\pi}{4}))$

A. **B.**

C. $-\frac{\pi}{4}$ **D.**

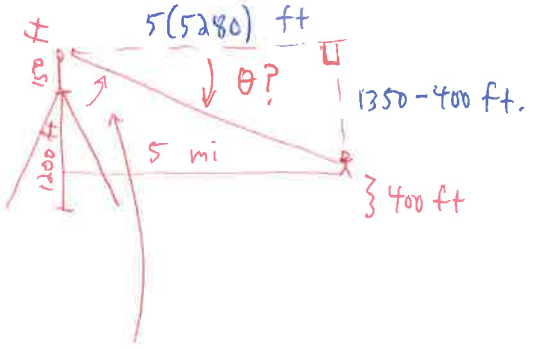
A **18.** Derive a triple angle formula: rewrite $\cos(3x)$ in terms of $\cos(x)$.

A. $4\cos^3(x) - 3\cos(x)$ **B.**

C. **D.**

Part IV: Short Answer [10 points each]
Show enough work. Clearly mark your final answers. Partial credit given when deserved.

19. A cell phone tower that is 150 ft tall is placed on top of a mountain that is 1200 ft above sea level. What is the angle of depression from the top of the tower to a cell phone user who is 5 horizontal miles away and 400 ft above sea level? Answer in degrees, rounded to two decimal places.



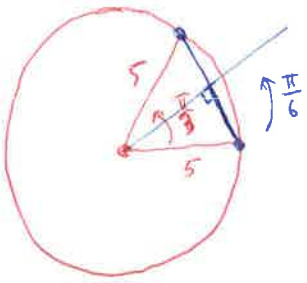
$\tan(\theta) = \frac{950}{26400}$

$\Rightarrow \theta = \arctan\left(\frac{950}{26400}\right)$

$\Rightarrow \theta \approx 2.06^\circ$ or 0.036 rad

this angle is the one many of you gave. It's $90^\circ - 2.06^\circ \approx 87.94^\circ$

20. Find the length of the sides of a regular hexagon inscribed in a circle of radius 5 cm.



$\sin(\pi/6) = \frac{\frac{1}{2}d}{5}$

so $d = 10\sin(\pi/6) = 10\left(\frac{1}{2}\right) = 5$ cm

so the sides have length $\boxed{5 \text{ cm}}$ each.

$6\left(\frac{\pi}{3}\right) = 2\pi$

so $\frac{\pi}{3}$ is $\frac{1}{6}$ of the unit circle.

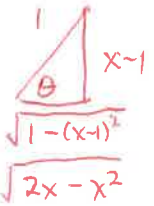
21. Verify the identity: $1 + \cos(10x) = 2\cos^2(5x)$.

$1 + \cos(10x) = 1 + 2\cos^2(5x) - 1$ double-angle formula!

$= 2\cos^2(5x)$ □

22. Write an algebraic expression that is equivalent to $f(x) = \sec(\arcsin(x-1))$. What is the domain of the new expression?

$\theta = \arcsin(x-1) \Rightarrow x-1 = \sin(\theta)$



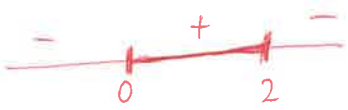
$= \sqrt{2x - x^2}$

$\sec(\theta) = \frac{1}{\sqrt{2x-x^2}}$ so $\boxed{f(x) = \frac{1}{\sqrt{2x-x^2}}}$

so [illegible] whoops :-/

domain: $2x - x^2 \geq 0$

$x(2-x) \geq 0$



so $\boxed{\text{dom}(f) = (0,2)}$

open