Vectors

- pts P, Q, vector $\overrightarrow{PQ}$
- length = magnitude

$\langle 5, 3 \rangle$ $\langle 1, 1 \rangle$

$(3, 1)$ $(-1, -1)$

- Component form
- Equivalence classes.
- unit vectors.
- convert $\langle 3, -4 \rangle$ into a unit v. in same direction

Ex. $P(4, -7)$, $Q(-1, 5)$

do the same.

Fact: $\|\vec{v}\| = 0$ iff $\vec{v} = \vec{0} = \langle 0, 0 \rangle$.

- How to add graphically, component-wise
- How to scalar mult. graphically, componentwise.

Properties:

$\vec{u}, \vec{v}, \vec{w}$ vectors, c, d scalars.

1. $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
2. $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
3. $\vec{u} + \vec{0} = \vec{u}$
4. $\vec{u} + (-\vec{u}) = \vec{0}$
5. $c(d\vec{u}) = (cd)\vec{u}$
6. $(c + d)\vec{u} = c\vec{u} + d\vec{u}$
7. $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$
8. $1(\vec{u}) = \vec{u}$, $0(\vec{u}) = \vec{0}$.
9. $\|c\vec{v}\| = |c| \, \|\vec{v}\|$

Ex. $\vec{u} = \langle 1, -4 \rangle$, $\vec{v} = \langle 3, 2 \rangle$

Find $\vec{u} + \vec{v}$, $\vec{u} - \vec{v}$, $2\vec{u} - 3\vec{v}$.

Ex. Find a vector w/ magnitude $\|w\| = 5$ in the direction of $\vec{v} = \langle -2, 3 \rangle$

standard basis vectors

$\vec{i} = \langle 1, 0 \rangle$

$\vec{j} = \langle 0, 1 \rangle$

$\vec{v} = \langle v_1, v_2 \rangle = v_1 \vec{i} + v_2 \vec{j}$.

linear combinations.

Ex. $\vec{u} = -3\vec{i} + 8\vec{j}$ $\quad \vec{v} = 2\vec{i} - \vec{j}$

Find $2\vec{u} - 3\vec{v}$.

Direction Angles.

$\vec{u} = \langle x, y \rangle \quad r = \sqrt{x^2 + y^2}$

spse $r = 1$ so $\vec{u}$ lies on unit circle

then $x = \cos\theta$ and $y = \sin\theta$

$\vec{u} = \langle \cos\theta, \sin\theta \rangle = (\cos\theta)\vec{i} + (\sin\theta)\vec{j}$.

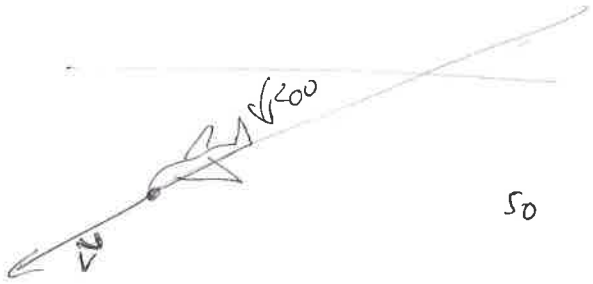
If $\|\vec{v}\| \neq 1$, then $\vec{v} = \|\vec{v}\| \left((\cos\theta)\vec{i} + (\sin\theta)\vec{j} \right)$

$\tan\theta = \frac{y}{x} \Rightarrow \theta = \arctan(y/x)$, in the correct Q.

Ex. Find θ for $\vec{u} = 3\vec{i} + 3\vec{j}$

$\vec{v} = 3\vec{i} - 4\vec{j}$. ($53.13°$ or $306.87°$)

Ex. Velocity of an airplane: descending at 150 mph, $20°$ below horizontal.



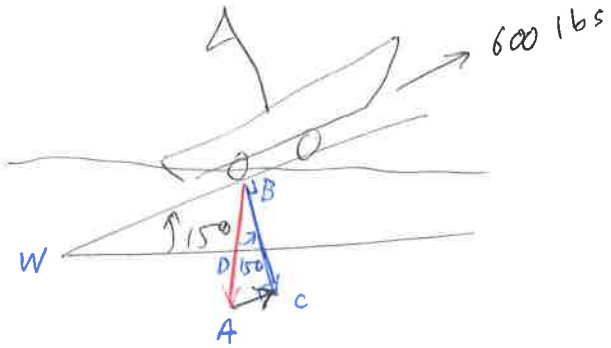
So $\|\vec{v}\| = 150$

$\theta = 20°$

Find $\vec{v}$.

get $\vec{v} = \langle -140.96, -51.30 \rangle$

Ex. 600 lb force req. to pull a boat up a ramp at 15°



$\|\vec{BA}\|$ = force due to gravity = combined weight of boat and trailer.

$\|\vec{BC}\|$ = force against ramp

$\|\vec{AC}\| = 600$ lbs

$$\sin 15^\circ = \frac{\|\vec{AC}\|}{\|\vec{BA}\|} = \frac{600}{\|\vec{BA}\|}$$

$$\|\vec{BA}\| = \frac{600}{\sin 15^\circ} \approx 2319 \text{ lbs.} \quad \underline{\text{WORD}}.$$