1.5: Graphs of Sine and Cosine

We begin with sine:

recall $\sin(\theta) = y$. We may think of this as a function that takes in a number between $(-\infty, \infty)$ and gives back a number between $[-1, 1]$.

For the sake of drawing the graph, put $\theta = x$.

The function we want to graph is now:

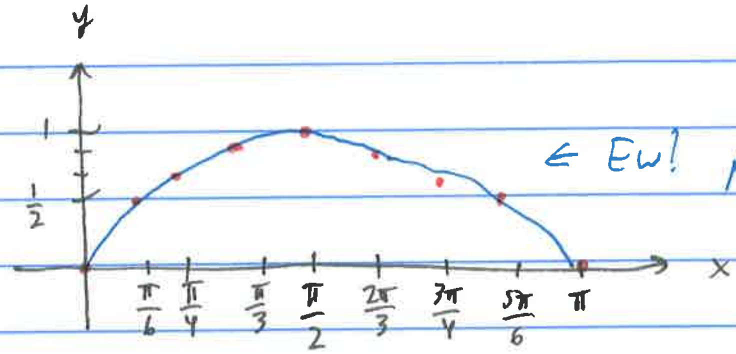
$$y = \sin(x)$$

Let's make a table of values using the unit circle for help

x	$y = \sin(x)$
0	0
$\pi/6$	$1/2$
$\pi/4$	$\sqrt{2}/2$
$\pi/3$	$\sqrt{3}/2$
$\pi/2$	1
$2\pi/3$	$\sqrt{3}/2$
$3\pi/4$	$\sqrt{2}/2$
$5\pi/6$	$1/2$
π	0
$\vdots$	$\vdots$

etc.

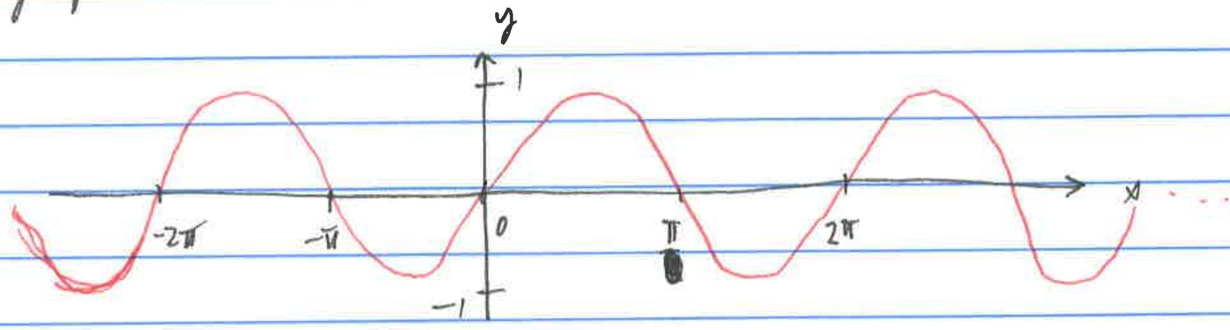
Plotting these points we get the following picture. The points should all be connected by a smooth curve.



← Ew! pretend that looks like " ".

Sorry :(

The ~~relations~~ pattern of these points continues so that the graph of sine looks like:



Every peak and valley should be at the same height. and all of the "waves" should look the same

The <u>period</u> of this curve is 2π: it makes one full "wave" in that time.

The <u>amplitude</u> is 1: that's the height of the peaks.

The general equation looks like:

$a, b, c, d \in \mathbb{R}$

$$\boxed{y = a \sin(bx + c) + d}$$

$\boxed{\text{Graph these using Wolfram|Alpha}}$

let's see what affect those numbers have.

Ex. $y = 3\sin(x)$

Amplitude is now 3

Ex. $y = -2\sin(x)$

Amplitude is now $|-2| = 2$.

So the amplitude of $y = a\sin(bx + c) + d$ is $|a|$.

Ex. $y = \sin(x) + 3$

Shifts graph up 3.

Ex. $y = \sin(x) - \pi$

Shifts down π units.

Ex. $y = \sin(x + \pi)$

shifts left π

Ex. $y = \sin\left(x - \frac{\pi}{2}\right)$

Shifts right $\pi/2$.

Ex. $y = \sin(2x)$

this changes the period to π

b is called the frequency of the graph. it tells you how many waves happen in a single (usual) period of 2π.

Ex. $y = \sin(2x+1) = \sin\left(2\left(x+\frac{1}{2}\right)\right)$

so the freq is still 2, but the left shift is really only $\frac{1}{2}$.

(*) This is a key idea to keep in mind!

Ex. $y = -\sin(2x - \pi) + 2$

Find period, amplitude, freq, shift.

(*) Do all of this for cosine.