Back to section 1.3 briefly

Fundamental Trig Identities:

Reciprocal Ids:

$\sec\theta = \frac{1}{\cos\theta}$ $\qquad$ $\cos\theta = \frac{1}{\sec\theta}$

$\csc\theta = \frac{1}{\sin\theta}$ $\qquad$ $\sin\theta = \frac{1}{\csc\theta}$

$\cot\theta = \frac{1}{\tan\theta}$ $\qquad$ $\tan\theta = \frac{1}{\cot\theta}$

Quotient Ids:

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\qquad$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

(*) Verify Ids on pg. 18 of notes.

Pythagorean Ids:

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \cot^2\theta = \csc^2\theta$$
$$\tan^2\theta + 1 = \sec^2\theta$$

Ex. Let θ be acute and $\sin\theta = 0.6$. Find the values of $\cos\theta$ and $\tan\theta$.

$\sin^2\theta + \cos^2\theta = 1$

$(0.6)^2 + \cos^2\theta = 1$

$\cos^2\theta = 1 - 0.36$

$\cos^2\theta = 0.64$

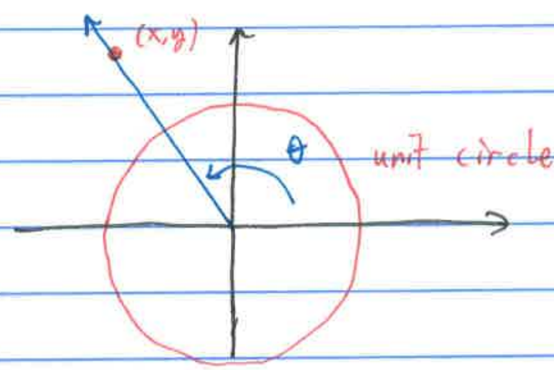
$\cos\theta = +\sqrt{0.64}$ + since θ is acute

$\cos\theta = 0.8$

Then $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{0.6}{0.8} = \frac{3}{4} = 0.75$

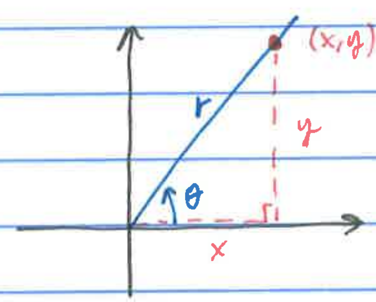
Section 1.4

Let θ be an angle in standard position and (x,y) a point on the terminal side, but not necessarily on the unit circle:



We should be able to find the trig function values for θ by using the point (x,y).

Ex. θ in QI.



by the distance formula or Pythagorean Thm,

$$r^2 = x^2 + y^2 \quad \text{so}$$

$$r = \sqrt{x^2+y^2} \neq 0 \quad \text{as long as } (x,y) \neq (0,0).$$

We can then use the right triangle def'ns to write the def'ns of the trig functions.

We get:

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x},\ x \neq 0$$
$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y},\ y \neq 0$$
$$\tan\theta = y/x,\ x \neq 0 \qquad \cot\theta = \frac{x}{y},\ y \neq 0$$

(*) Notice that if we use a point in the plane then the $\pm$ signs on x and y will give us the proper sign on the trig function values.

This fixes the main problem with the right triangle def'ns.

Ex. $\tan\theta = \frac{-5}{4}$ $\cos\theta > 0$. Find all 6.

$\cos\theta > 0$ means $x > 0$

Therefore $x = 4$ and $y = -5$

$r = \sqrt{16 + 25} = \sqrt{41}$

Put these together to find the rest.

$\cos\theta = \frac{x}{r} = \frac{4}{\sqrt{41}}$

$\sin\theta = \frac{y}{r} = \frac{-5}{\sqrt{41}}$ ... etc.

$\tan\theta = -5/4$

(*) Don't forget to divide by r!

Ex. $(x,y) = (-3,4)$

Find $\sin\theta$, $\cos\theta$, $\tan\theta$.

Ex. θ in QII $\sin\theta = \frac{1}{3}$. Find the rest.

Ex. Find two solutions of the equation in $0 \le \theta < 2\pi$.

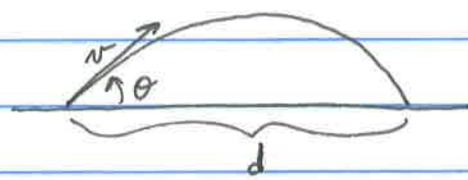
a). $\sin\theta = -\frac{1}{2}$

b). $\sec\theta = 2$

c) $\cot\theta = -1$

Ex. the horizontal distance traveled by a projectile with initial speed v ft/sec is

$$d = \frac{v^2}{32}\sin(2\theta)$$



A golf ball is hit w/ initial speed 100 ft/sec at an angle of $30°$. Find the distance.

$$d = \frac{100^2}{32}\sin(2(30°)) = \frac{10000}{32}\sin(60°) = \frac{10000}{32}\left(\frac{\sqrt{3}}{2}\right)$$

$$= 270.63 \text{ ft.}$$

45°? $\frac{10,000}{32}(\sin 90°) = \frac{10,000}{32} = 312.5$ ft.

60°? $\frac{10000}{32}\sin(120°) = \frac{10000}{32}\sin 60° = 270.63$ ft.

75°? $\frac{1000}{32}\sin(150) = \frac{10000}{32}(\sin 30°) = \frac{10000}{32}\cdot\frac{1}{2} = 156.25$ ft.

"Too high, too high!"

__Next:__ graphs of sine and cosine.