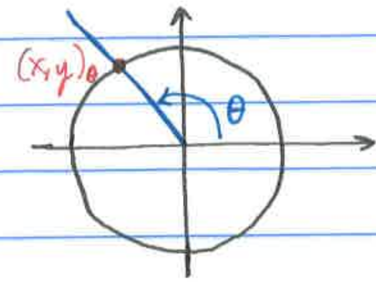
1.2: Trigonometric Functions

: first skip ahead to p.13 to review functions, then come back to this page.

Unit circle definitions



For any angle θ, there is a unique point on the unit circle that corresponds to it. Call this point $(x,y)_\theta$ or just (x,y).

We define the trigonometric functions in the following way. They all take in an angle as their argument and give back a real number, so long as they are defined for that angle.

Defn. Let θ be an angle and $(x,y)_\theta$ the corresponding point on the unit circle. Then

$$(*)\quad \boxed{\begin{array}{ll} \cos(\theta) := x & \sec(\theta) := \frac{1}{x} \\ \sin(\theta) := y & \csc(\theta) := \frac{1}{y} \\ \tan(\theta) := y/x,\ x \neq 0 & \cot(\theta) := x/y \end{array}} \quad \begin{array}{l} x \neq 0 \\ y \neq 0 \\ y \neq 0 \end{array}$$

cos = cosine   sec = secant

sin = sine   csc = cosecant

tan = tangent   cot = cotangent

Brief review of functions.

Need more? Look at sections P.5 and P.8.
Later P.6, P.7, P.9, and P.10 will be very relevant.
We'll review them a little, as necessary.

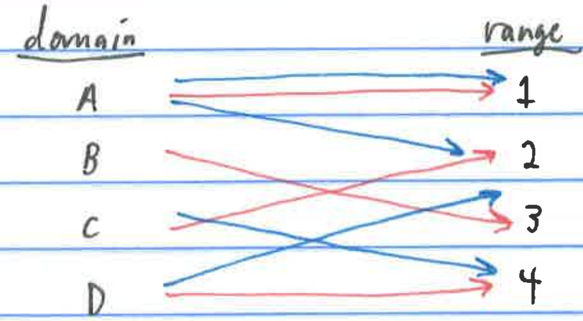
A <u>function</u> has three parts: domain, range, assignment between them.

the <u>domain</u> of a function is its set of input values.
"What are you allowed to put in?"

the <u>range</u> is the set of all output values.
"What do you get back?"

the assignment tells you how to get from the domain to the range. It's usually given as an algebraic formula; e.g., $f(x) = x^2$.
We can visualize the assignment as a collection of arrows.

<u>ex</u>.



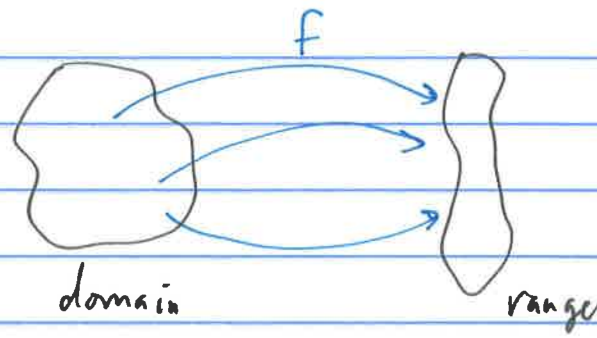
f is a function

f is <u>not</u> a function

so what's the difference?

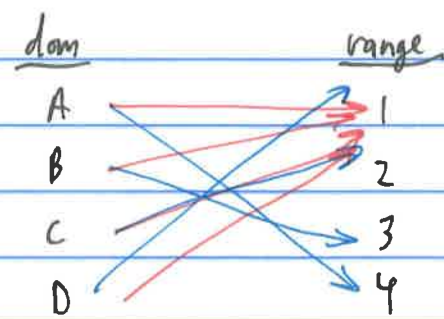
Precice definition of a function:

$\boxed{\text{Def}^n}$ A function is a relation between two sets called the domain and the range such that for every x in the domain, there is exactly one corresponding y in the range.



In terms of arrows, this means that every element in the domain must have exactly one arrow coming out.

Ex.



both are functions.

Ex. If a function is only given by its assignment, find the domain:

$$y = f(x) = \frac{x}{1-x^2}$$

Ask: what can go wrong? $1-x^2 \neq 0 \Rightarrow x \neq \pm 1$

So the domain is everything except ± 1: $\boxed{(-\infty,-1) \cup (-1,1) \cup (1,\infty)}$

Ex. Find the domain of $f(x) = \sqrt{x-4}$

$$x - 4 \geq 0$$
$$x \geq 4$$
$$\Rightarrow \text{ domain is } [4, \infty).$$

... Now back to the trig functions

Let's find the domains of the trig functions.

Every θ determines a point on the unit circle, so sin and cos are defined for all θ.

tangent and secant and defined except when $x = 0$.
$x = 0$ when $\theta = \pi/2$ or $3\pi/2$, or $2n\pi$ plus these angles.

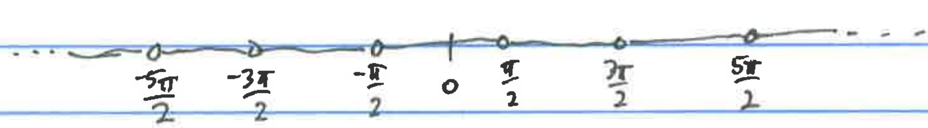
So for tan and sec, $\theta \neq \frac{\pi}{2} \pm 2\pi n$, $n = 0, 1, 2, \dots$
$\theta \neq \frac{3\pi}{2} \pm 2\pi n$, $n = 0, 1, 2, \dots$

but θ can be anything else.

It turns out that we can combine these to write:

$$\theta \neq \frac{\pi}{2} \pm \pi n, \quad n = 0, 1, 2, \dots$$

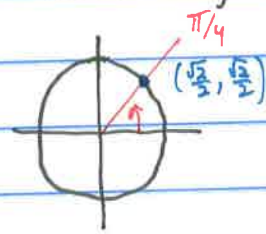
The intervals that they are defined on look like:



RE **Exercise for you:** show that cosecant and cotangent have $\theta \neq \pm n\pi$ $n = 0, 1, 2, \ldots$ and draw a few of the intervals.

Now let's find some values of these trig functions for special angles. (i.e., unit circle angles).

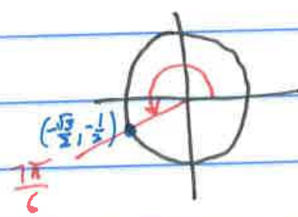
Ex. $\sin(\pi/4)$



$\sin(\pi/4)$ is the y-value of the point $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = (x,y)_{\frac{\pi}{4}}$.

There fore, $\boxed{\sin(\pi/4) = \frac{\sqrt{2}}{2}}$

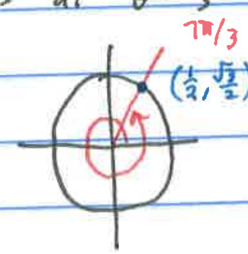
Ex. $\cos\left(\frac{7\pi}{6}\right)$



$(x,y)_{\frac{7\pi}{6}} = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

so $\cos\left(\frac{7\pi}{6}\right) = \frac{-\sqrt{3}}{2}$

Ex. Find the values of all 6 trig functions at $\theta = \frac{7\pi}{3}$



$\frac{7\pi}{3} = \frac{6\pi}{3} + \frac{\pi}{3} = 2\pi + \frac{\pi}{3}$

so $(x,y)_{\frac{7\pi}{3}} = (x,y)_{\pi/3} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

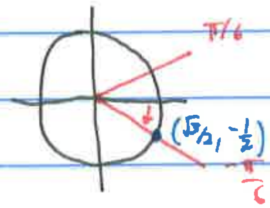
! $\left\{ \begin{array}{l} \cos(7\pi/3) = x = \frac{1}{2} \\ \sin(7\pi/3) = y = \sqrt{3}/2 \\ \tan(7\pi/3) = y/x = \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = \sqrt{3} \end{array} \right.$

$\sec\left(\frac{7\pi}{3}\right) = \frac{1}{x} = \frac{1}{1/2} = 2$

$\csc\left(\frac{7\pi}{3}\right) = \frac{1}{y} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cot\left(\frac{7\pi}{3}\right) = \frac{x}{y} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

Ex. Find all 6 trig functions for $\theta = -\pi/6$.



$$(x,y)_{-\frac{\pi}{6}} = \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

$\cos\left(-\frac{\pi}{6}\right) = x = \frac{\sqrt{3}}{2}$

$\sin\left(-\pi/6\right) = y = -\frac{1}{2}$

$\tan\left(-\pi/6\right) = y/x = \frac{-1/2}{\sqrt{3}/2} = -\frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\sec\left(-\frac{\pi}{6}\right) = \frac{1}{x} = \frac{2}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\csc\left(-\pi/6\right) = \frac{1}{y} = -2$

$\cot\left(-\pi/6\right) = \frac{x}{y} = \frac{\sqrt{3}/2}{-1/2} = \frac{\sqrt{3}}{2}\left(-\frac{2}{1}\right) = -\sqrt{3}$

Some vocabulary / properties:

- Cosine is an even function because $\cos(-\theta) = \cos\theta$.
- Sine is an odd function because $\sin(-\theta) = -\sin\theta$.

RE What about the other 4? [Hint: one is even, and the rest are odd.]

- All of the trig functions are periodic.

In other words, there exists a positive number c such that $f(\theta + c) = f(\theta)$.
The smallest such c is called the period.

sin, cos, sec, csc have period 2π
tan, cot have period π
} We'll do some in class. You should verify the rest.