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Target Simulations

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The system of equations of compressible magnetohydrodynamics:

an example of a coupled hyperbolic – parabolic/elliptic subsystems

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u})$$

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) \mathbf{u} = -\nabla P + \mu \Delta \mathbf{u} + \boxed{\frac{1}{c} (\mathbf{J} \times \mathbf{B})}$$

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla \right) e = -P \nabla \cdot \mathbf{u} + \boxed{\frac{1}{\sigma} \mathbf{J}^2}$$

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times \left(\frac{c^2}{4\pi\sigma} \nabla \times \mathbf{B} \right)}$$

$$P = P(\rho, e), \qquad \boxed{\nabla \cdot \mathbf{B} = 0}$$

Constant in time magnetic field approximation (low magnetic Reynolds number)

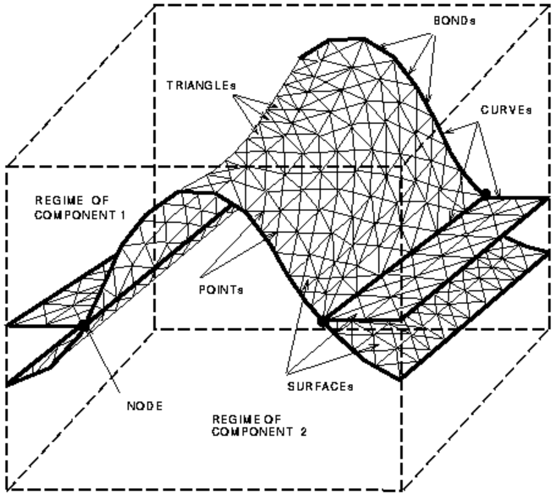
- The diffusion time of the magnetic field is of order 10 microseconds
- The eddy current induced magnetic field is negligible
- Therefore, the time evolution of B field can be neglected and the distribution of currents can be found by solving Poisson’s equation:

$$\mathbf{J} = \sigma\left(-\nabla\phi + \frac{1}{c}\mathbf{u}\times\mathbf{B}\right)$$

$$\Delta\phi = \frac{1}{c}\nabla\cdot\left(\mathbf{u}\times\mathbf{B}\right),$$

$$\text{with}\quad \left.\frac{\partial\phi}{\partial\mathbf{n}}\right|_{\Gamma} = \frac{1}{c}(\mathbf{u}\times\mathbf{B})\cdot\mathbf{n}$$

Numerical methods for the hyperbolic subsystem inmplemented in the *FronTier Code*



■ The FronTier code is based on front tracking. Conservative scheme.

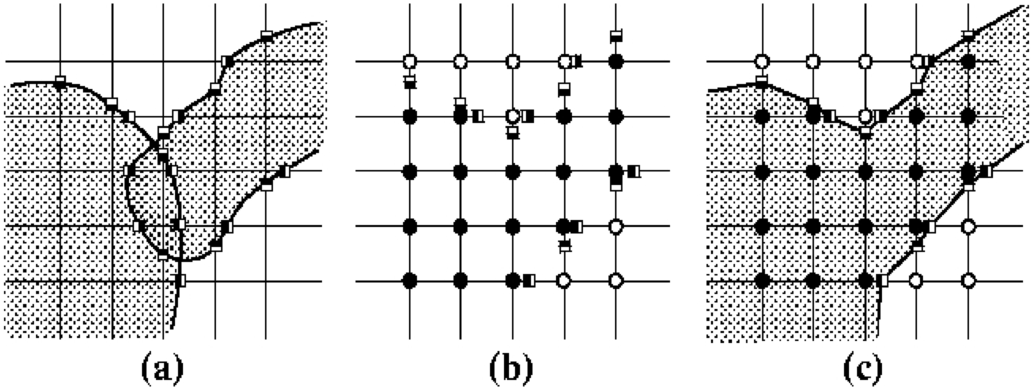
■ Front tracking features include the absence of the numerical diffusion across interfaces. It is ideal for problems with strong discontinuities.

■ Away from interfaces, FronTier uses high

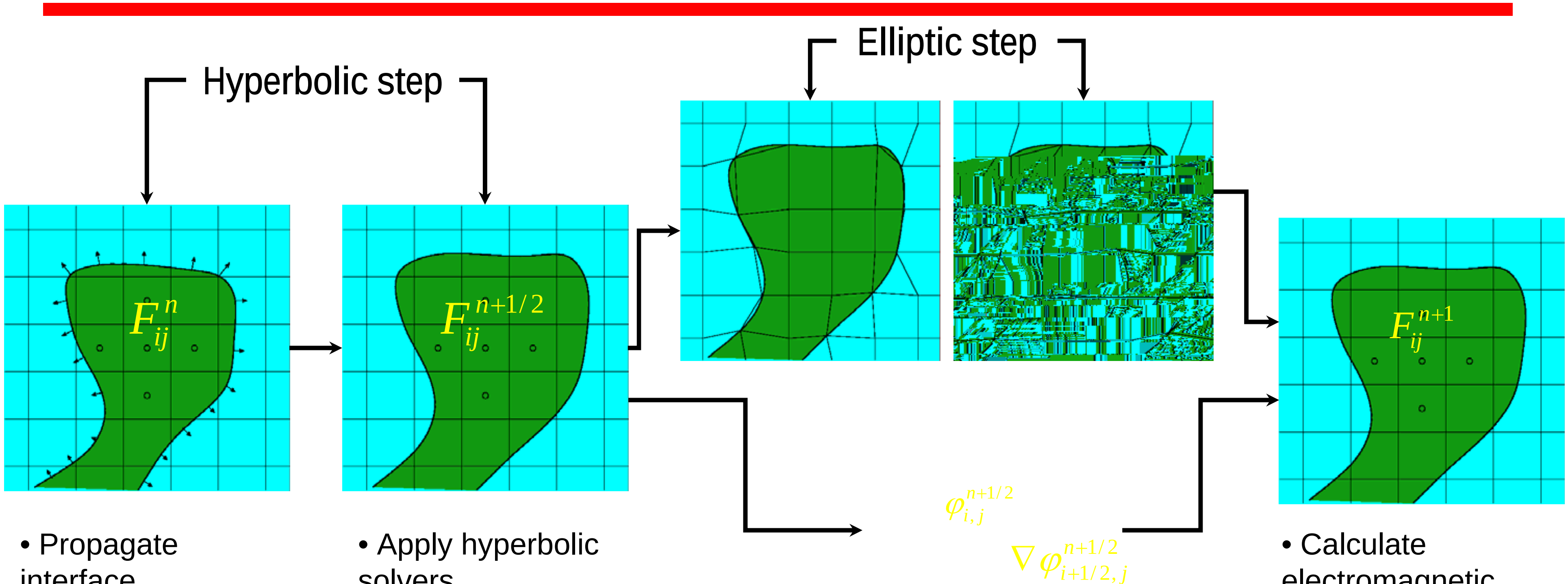
resolution (shock capturing) methods

■ FronTier uses realistic EOS models:

- SESAME
- Phase transition support



FronTier-MHD numerical scheme



- Propagate interface
- Untangle interface
- Update interface states

- Apply hyperbolic solvers
- Update interior hydro states

- Generate finite element grid
- Perform mixed finite element discretization

or

- Perform finite volume discretization
- Solve linear system using fast Poisson solvers

- Calculate electromagnetic fields
- Update front and interior hydro states

Low resolution run with dynamic cavitation. Energy deposition is 80 J/g

Initial density

Density at 3.5 microseconds

Initial pressure is 16 Kbar

Pressure at 3.5 microseconds

Density at 620 microseconds

Analysis of previous simulations

Positive features

• Qualitatively correct evolution of the jet surface due to the proton energy deposition

• Stabilizing effect of the magnetic field

Negative features

• Discrepancy of the time scale with experiments

• Absence of cavitation in mercury

• The growth of surface instabilities due to **unphysical** oscillations of the jet surface interacting with shock waves

• 2D MHD simulations do not explain the behavior of azimuthal modes

Conclusion

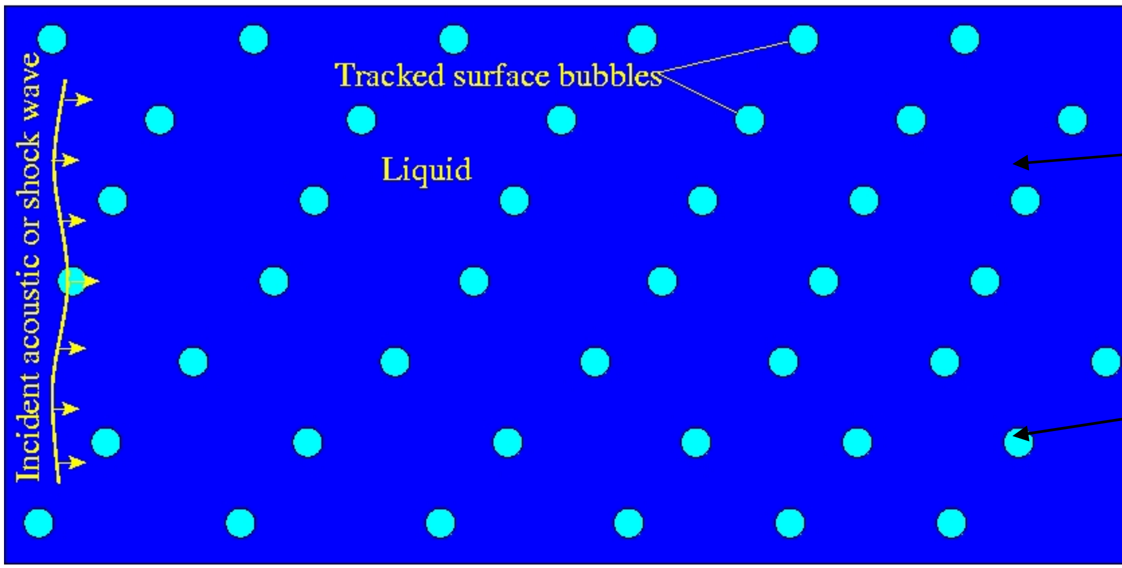
• Cavitation is very important in the process of jet disintegration

• There is a need for cavitation models/libraries for the FronTier code

• 3D MHD simulations are necessary

We have developed two approaches for cavitating and bubbly fluids

- **Direct numerical simulation method:** Each individual bubble is explicitly resolved using FronTier interface tracking technique.



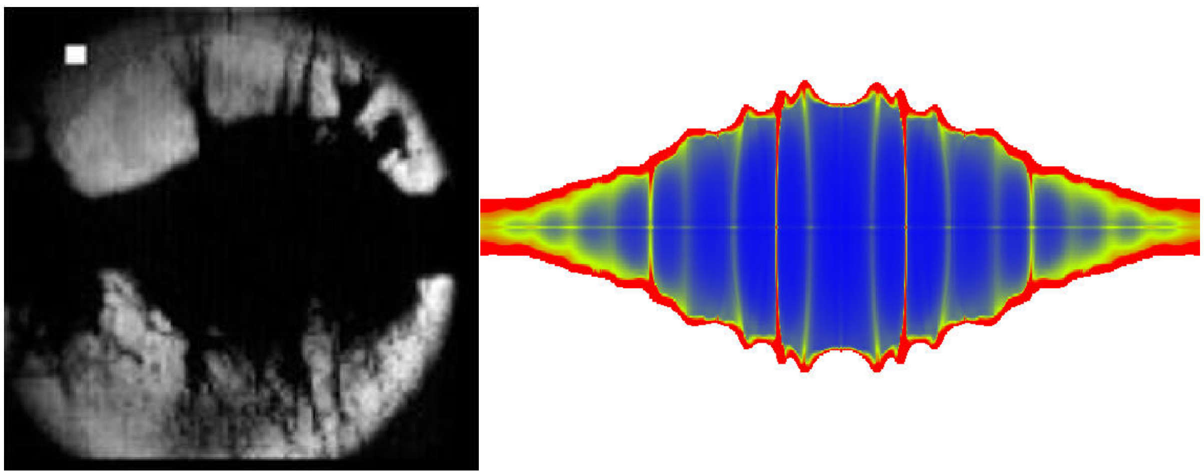
Stiffened Polytropic EOS for liquid

Polytropic EOS for gas (vapor)

- **Homogeneous EOS model.** Suitable average properties are determined and the mixture is treated as a pseudofluid that obeys an equation of single-component flow.

Homogeneous two phase EOS model

- Applicable to problems which do not require resolving of spatial scales comparable to the distance between bubbles.
- Accurate (in the domain of applicability) and computationally less expensive.
- Correct dependence of the sound speed on the density (void fraction).
- Enough input parameters to fit the sound speed to experimental data.



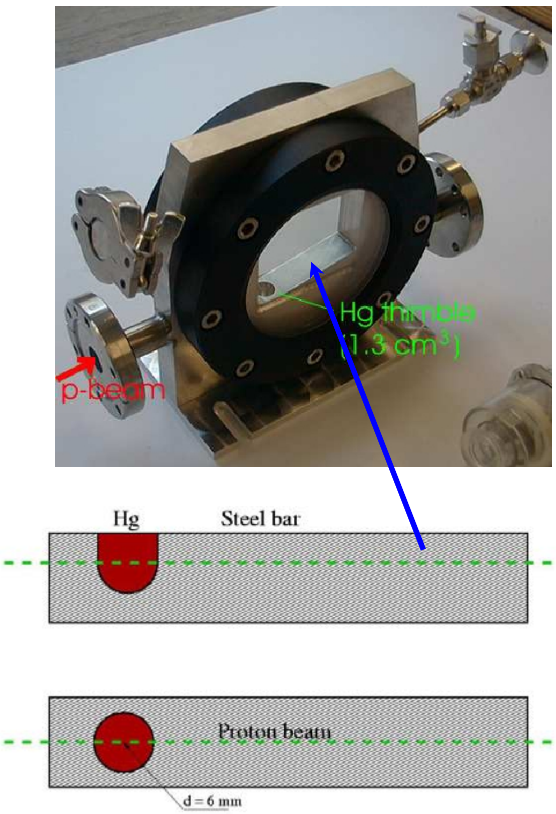
Experimental image (left) and numerical simulation (right) of the mercury jet.

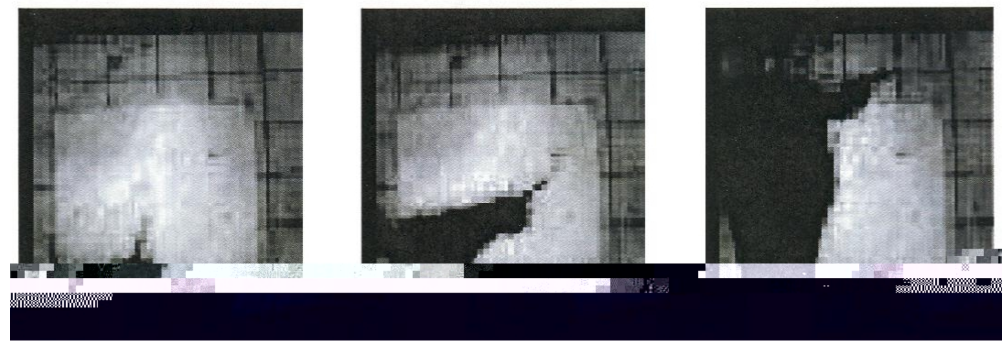
- Mercury splash is a combined effect of the expansion of the two-phase domain and the growth of Richtmyer-Meshkov (RM) surface instabilities.

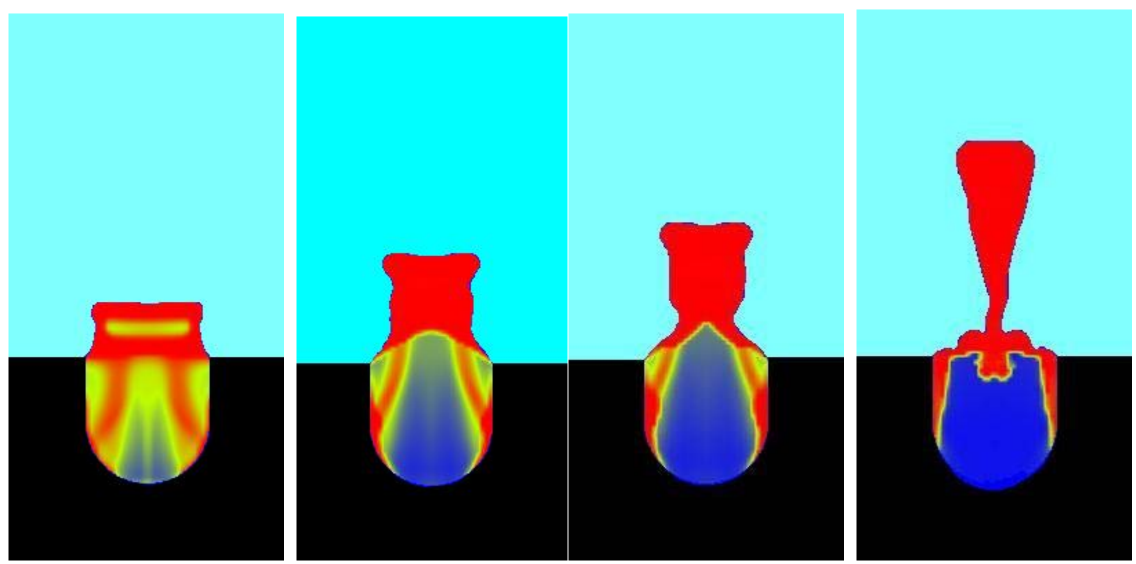
- Homogeneous EOS model does not resolve small scales (RM instabilities)

Numerical simulation of mercury thimble experiments

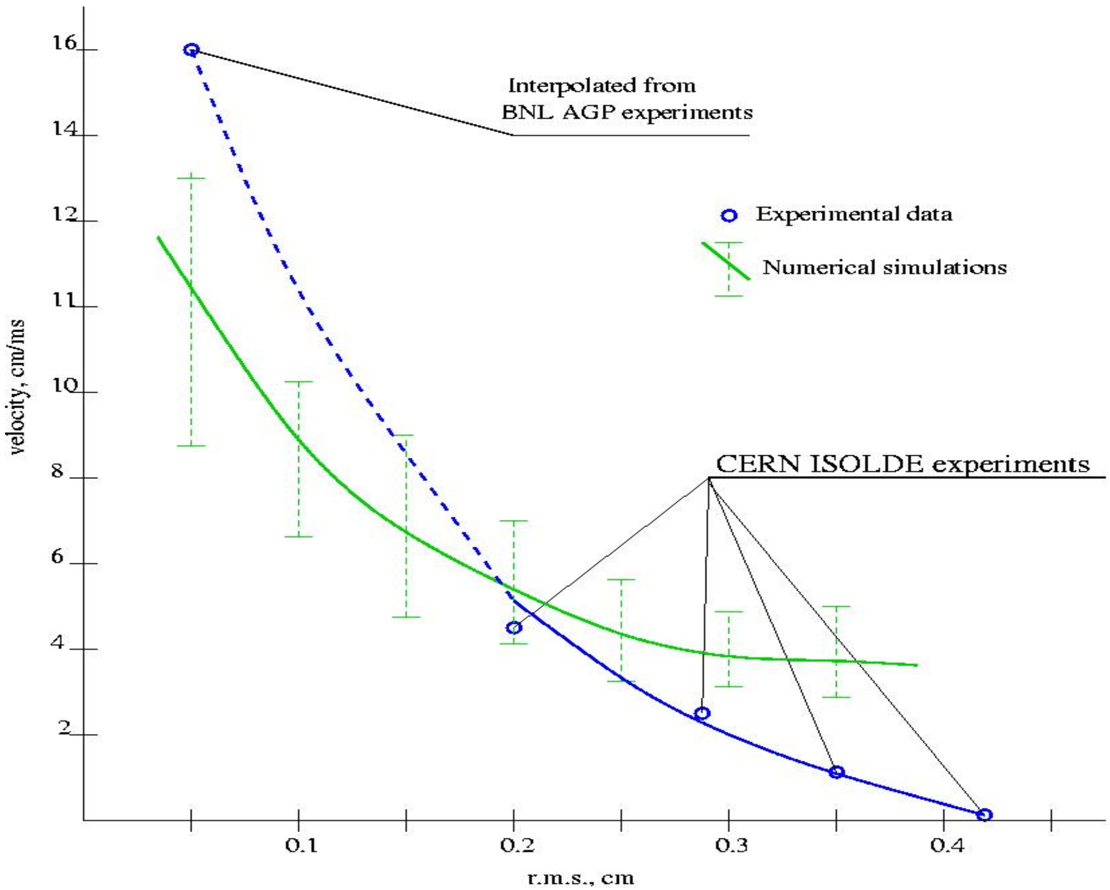
Evolution of the mercury splash due to the interaction with a proton beam (beam parameters: 24 GeV, $3.7*10^{12}$ protons). Top: experimental device and images of the mercury splash at 0.88 ms, 1.25 ms, and 7 ms. Bottom: numerical simulations using the FronTier code and analytical isentropic two phase equation of state for mercury.







Velocity as a function of the r.m.s. spot size



Experimental data: insufficient optical resolution to observe fine structure of RM instabilities. But RM instabilities are present.

• RM instabilities depend on the strength of the shock; the pressure wave has higher gradient at low r.m.s. (more like shock).

• Numerical data: mercury splash is due to the expansion of the two-phase domain. Homogeneous EOS model does not resolve small spatial scales (RM instabilities).

Features of the Direct Method

- Accurate description of multiphase systems limited only to numerical errors.

- Resolves small spatial scales of the multiphase system

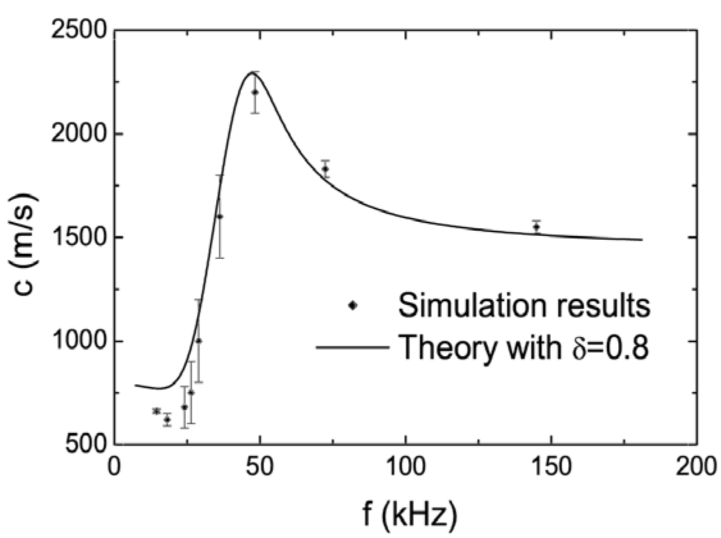
- Accurate treatment of drag, surface tension, viscous, and thermal effects.

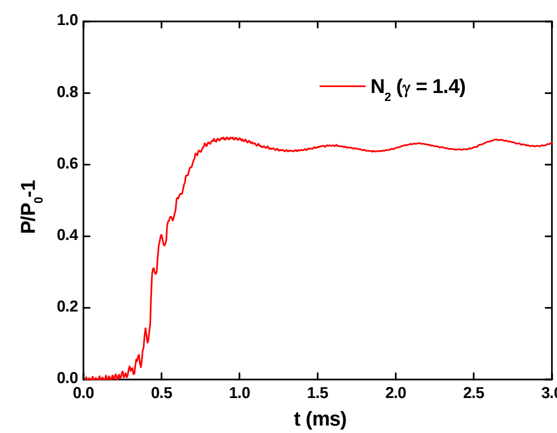
- Accurate treatment of the mass transfer due to phase transition (implementation in progress).

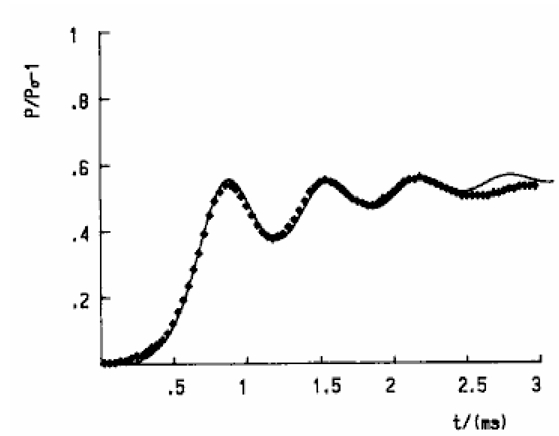
- Models some non-equilibrium phenomena (critical tension in fluids)

Validation of the direct method: linear waves and shock waves in bubbly fluids

- Good agreement with experiments (Beylich & Gülhan, sound waves in bubbly water) and theoretical predictions of the dispersion and attenuations of sound waves in bubbly fluids
- Simulations were performed for small void fractions (difficult from numerical point of view)
- Very good agreement with experiments of the shock speed
- Correct dependence on the polytropic index







Application to SNS target problem

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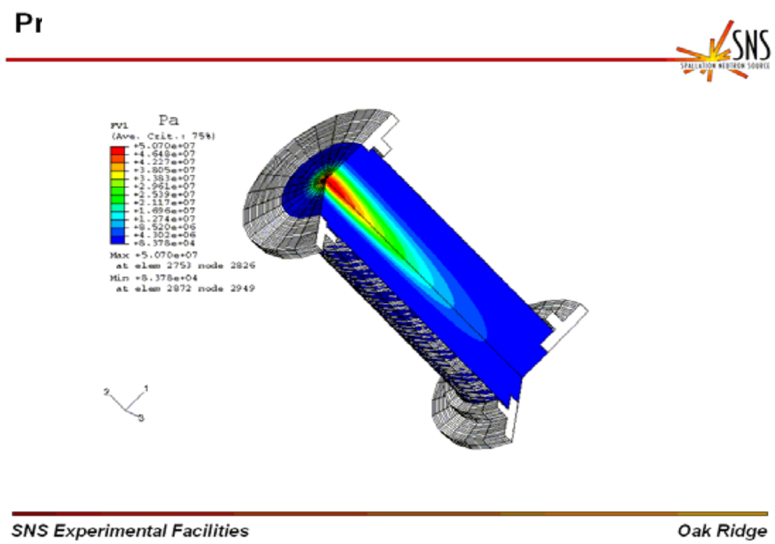
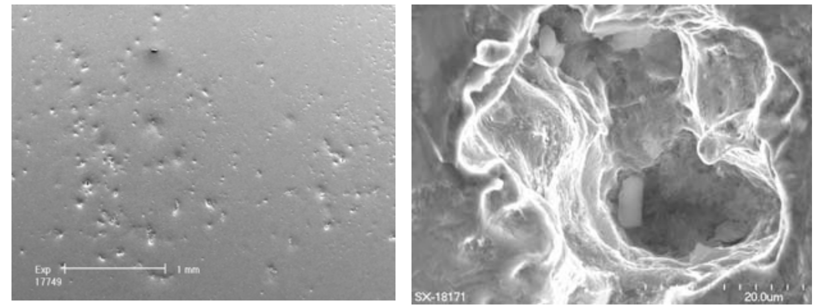
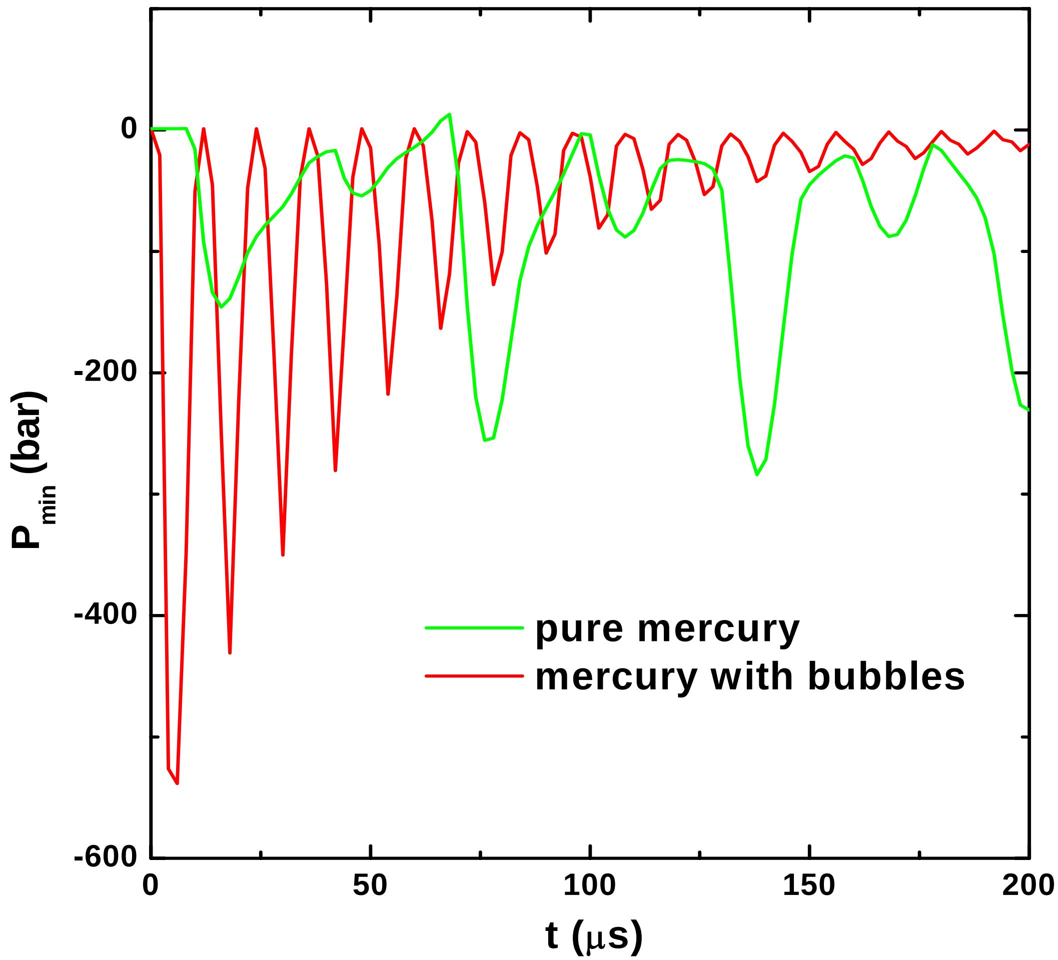




Left: pressure distribution in the SNS target prototype. Right: Cavitation induced pitting of the target flange (Los Alamos experiments)

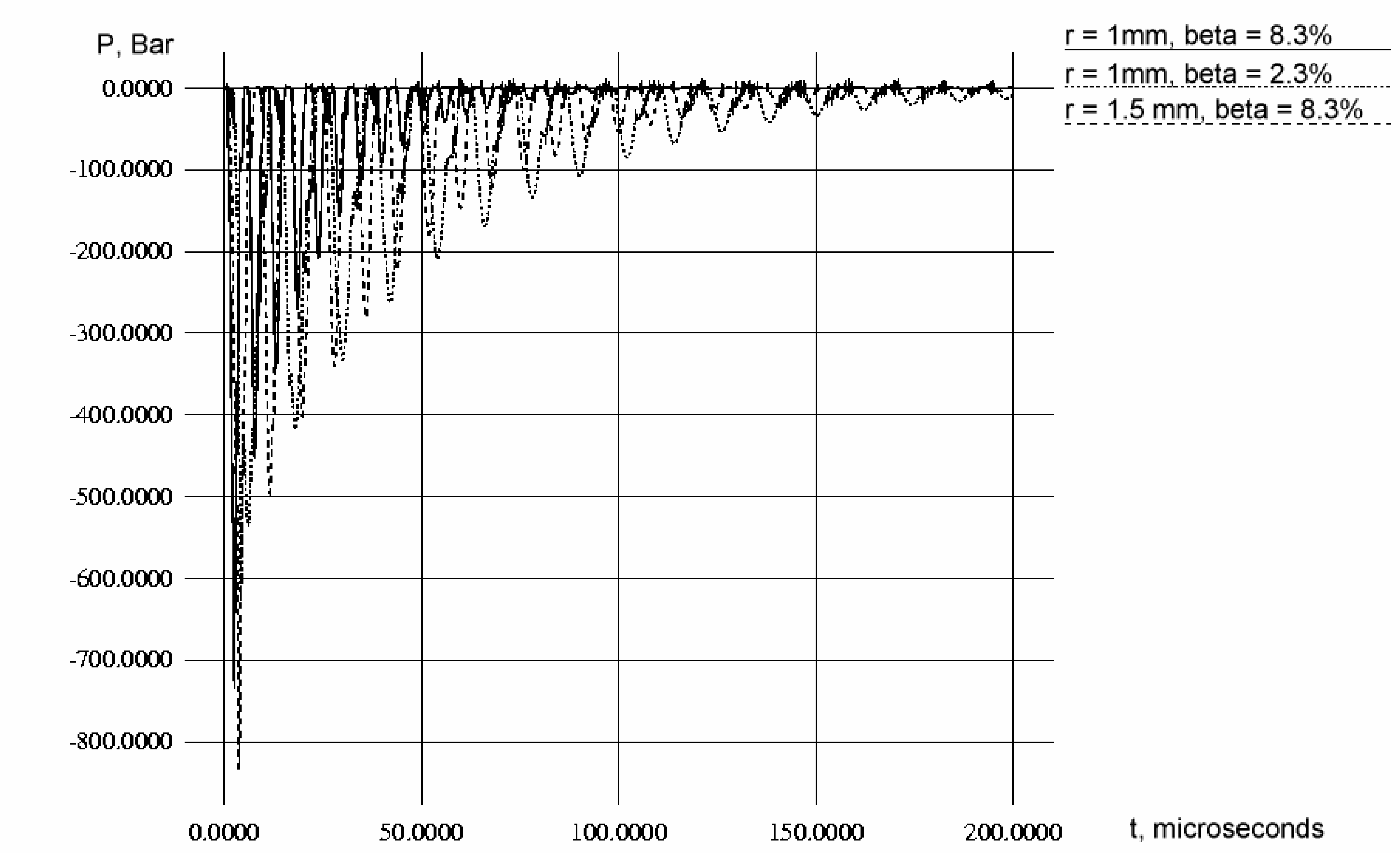
- Injection of nondissolvable gas bubbles has been proposed as a pressure mitigation technique.
- Numerical simulations aim to estimate the efficiency of this approach, explore different flow regimes, and optimize parameters of the system.

Application to SNS



Effects of bubble injection:

- Peak pressure decreases by several times.
- Fast transient pressure oscillations. Minimum pressure (negative) has larger absolute value.
- Cavitation lasts for short time


P, Bar
0.0000
-100.0000
-200.0000
-300.0000
-400.0000
-500.0000
-600.0000
-700.0000
-800.0000
0.0000
50.0000
100.0000
150.0000
200.0000
t, microseconds
r = 1mm, beta = 8.3%
r = 1mm, beta = 2.3%
r = 1.5 mm, beta = 8.3%

Dynamic cavitation

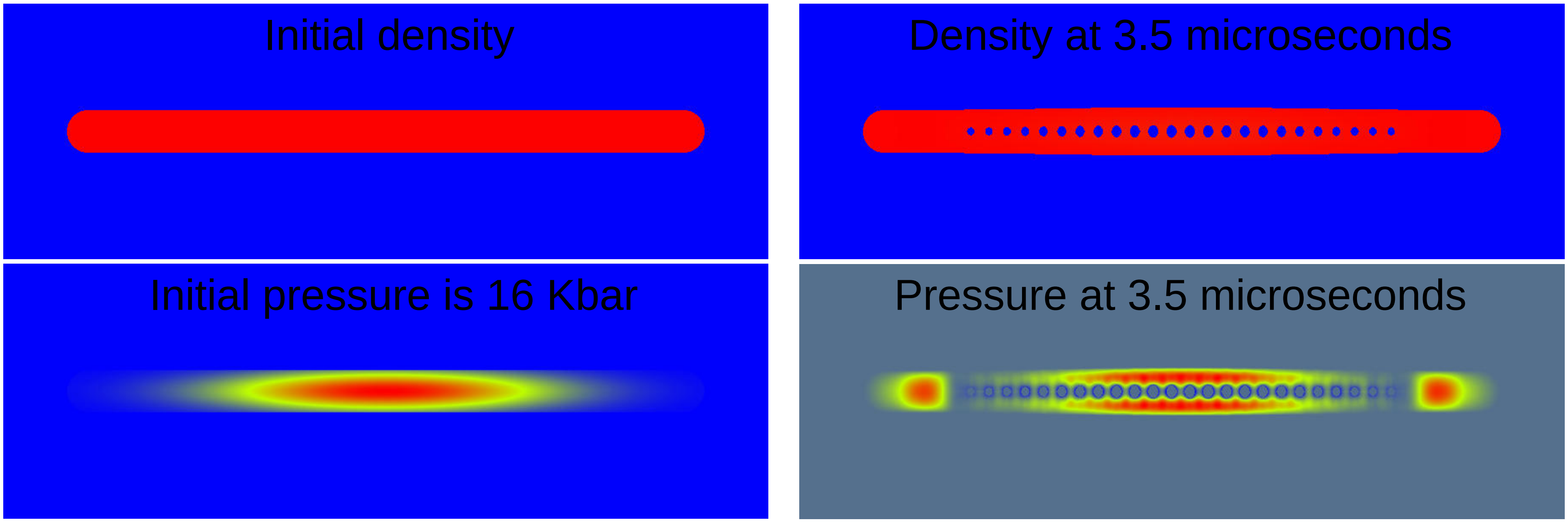
- A cavitation bubble is dynamically inserted in the center of a rarefaction wave of critical strength
- A bubbles is dynamically destroyed when the radius becomes smaller than critical. “Critical” radius is determined by the numerical resolution, not the surface tension and pressure.
- There is no data on the distribution of nucleation centers for mercury at the given conditions. Some theoretical estimates:

critical radius: $$R_C = \frac{2S}{\Delta P_C}$$

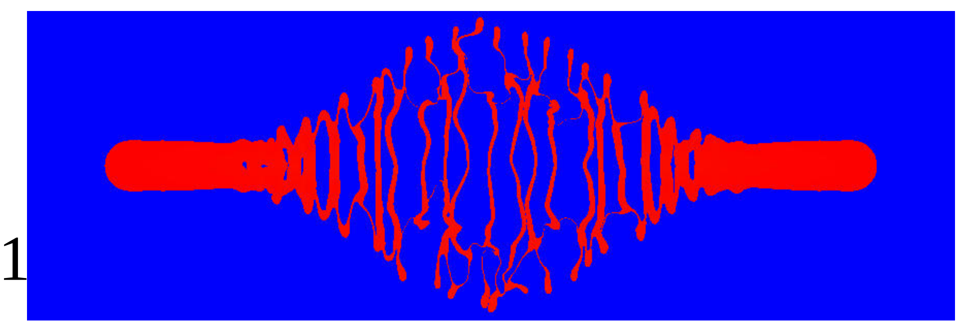
nucleation rate: $$J = J_0 e^{-Gb}, \quad J_0 = N\sqrt{\frac{2S}{\pi m}}, \quad Gb = \frac{W_{CR}}{kT}, \quad W_{CR} = \frac{16\pi S^3}{3\left(\Delta P_C\right)^2}$$

- A Riemann solver algorithm has been developed for the liquid-vapor interface.

Low resolution run with dynamic cavitation. Energy deposition is 80 J/g

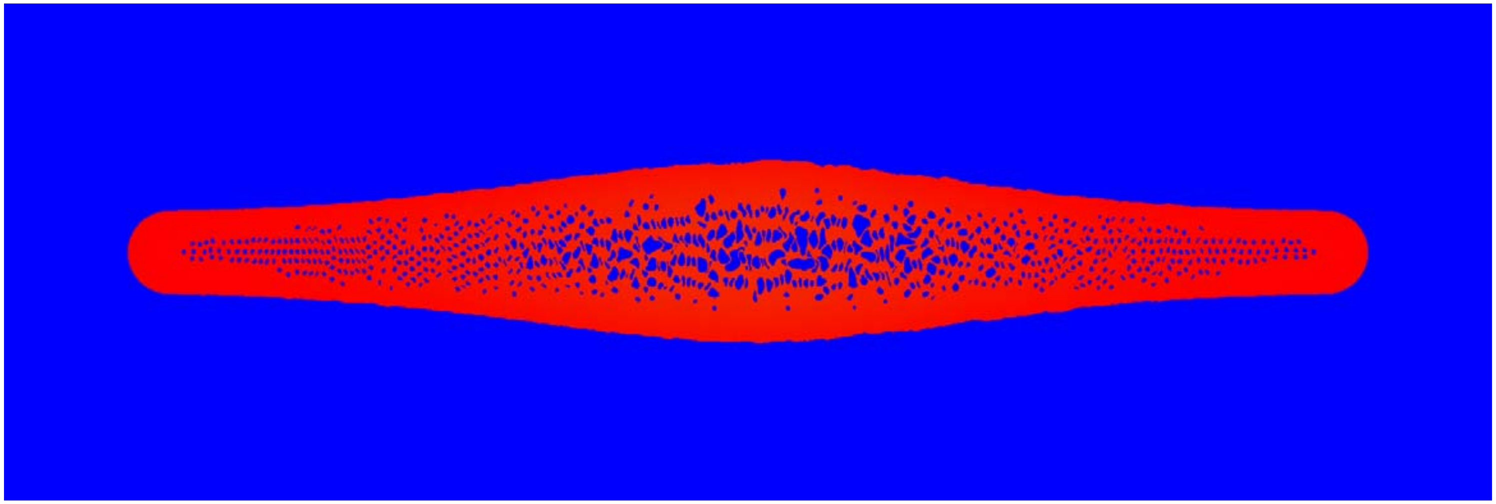


Density at 620 microseconds

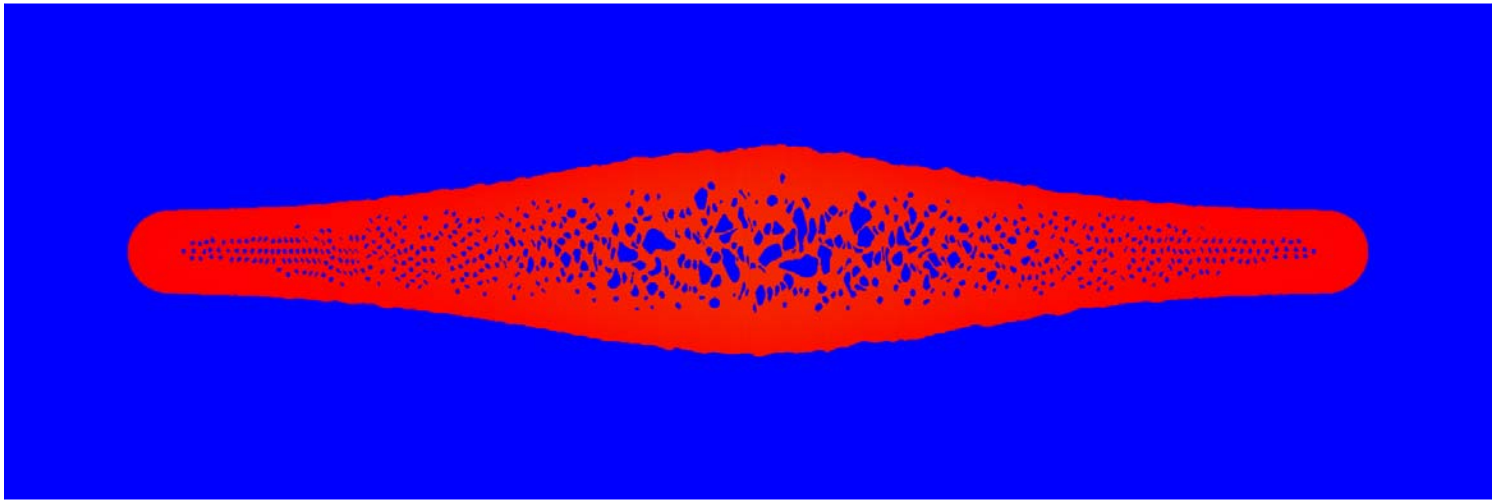


High resolution simulation of cavitation in the mercury jet

76 microseconds



100 microseconds



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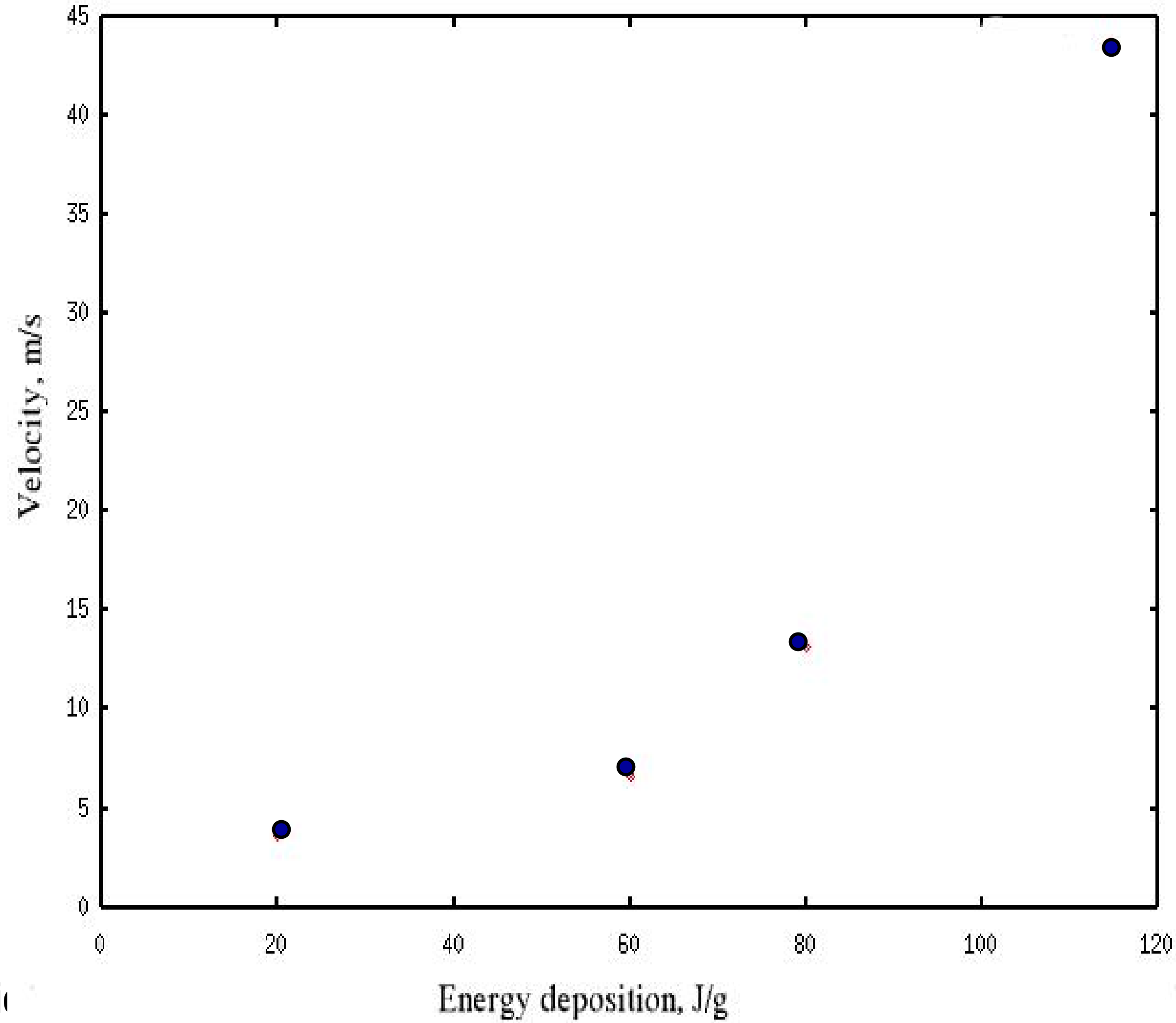
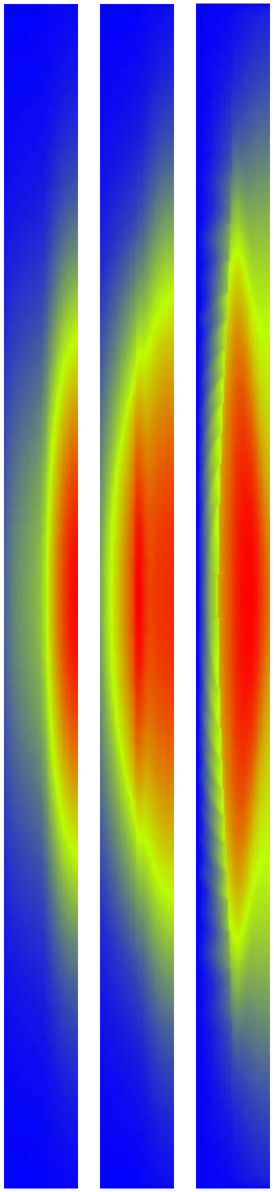
High resolution simulation of cavitation in the mercury jet



Interaction with multiple proton bunches.



• We have studies the wave structure and jet evolution after interaction with multiple proton bunches.

• Limit based on sound speed: 3.5 microseconds. Agrees with numerical experiments since there is no cavitation induced reduction of the sound speed during this time.

• Multiple pulses reduce surface velocity.

• Results depend on the beam r.m.s. spot size. Velocity change is smaller at large r.m.s.

• Multiple bunch beam itself has a similarity with the beam r.m.s. increase.

3D MHD simulations: summary of progress

- A new algorithm for 3D MHD equations has been developed and implemented in the code.

- The algorithm is based on the Embedded boundary technique for elliptic problems in complex domains (finite volume discretization with interface constraints).

- Preliminary 3D simulations of the mercury jet interacting with a proton pulse have been performed.

- Studies of longitudinal and azimuthal modes. Strong stabilization of longitudinal modes. Simulations showed that azimuthal modes are weakly stabilized (effect known as the flute instability in plasma physics)

- Combining of MHD and direct cavitation is necessary. The problem demands large computational resources.

Conclusions and Future Plans

- Two approaches to the modeling of cavitating and bubbly fluids have been developed
  - Homogeneous Method (homogeneous equation of state models)
  - Direct Method (direct numerical simulation)
- Simulations of linear and shock waves in bubbly fluids have been performed and compared with experiments. SNS simulations.
- Simulations of the mercury jet and thimble interacting with proton pulses have been performed using two cavitation models and compared with experiments.
- Both directions are promising. Future developments:
  - Homogeneous method: EOS based on the Rayleigh –Plesset equation.
  - Direct numerical simulations: AMR, improvement of thermodynamics, mass transfer due to the phase transition; distribution of cavitation centers.
  - Continue 3D simulations of MHD processes in the mercury target.
  - Coupling of MHD and cavitation models.