MATH555 Differential Equations

1. Find the explicit solution of the following initial value problem.

$$y' = \frac{2xy}{1+x^2}, \quad y(0) = 3.$$

2. Find the general solution of the following problem.

$$6(x+y)^2 + y^2 e^{xy} + 12x^3 + (e^{xy} + xye^{xy} + \cos y + 6(x+y)^2)y' = 0.$$

3. Find the general solution of the following problem.

$$y' + 2ty = 4t^3.$$

4. Use the provided solution to find the general solution to the equation.

$$(x-1)y'' - xy' + y = 0, \quad y_1(x) = e^x.$$

5. Solve the following initial value problem.

$$y'' - 2y' + 5y = 8\sin t - 4\cos t, \quad y(0) = 3, \quad y'(0) = 9.$$

6. Find the general solution of the following problem.

$$y'' + 18y' + 81y = 6e^{-9t}.$$

7. Find the general solution of the following problem.

$$y'' - 2y' + y = \frac{2e^t}{1+t^2}.$$

8. Use the power series method to determine the general solution to the equation.

$$(1-x^2)y'' - xy' + 4y = 0.$$

9. Use the power series method to determine the general solution to the equation.

$$2x^2y'' + 3xy' + (2x^2 - 1)y = 0.$$

10. Find the Laplace transform of

$$f(t) = u_5(t)(t-1)^2.$$

11. Find the Laplace transform of $f''(t)$ if

$$f(t) = te^{3t}\cos t.$$

12. Find the solution of the following initial value problem.

$$y'' + y = \delta(t-2\pi)\cos t; \quad y(0) = 0, \quad y'(0) = 1.$$

Solutions

1.
$$y(x) = 3(1+x^2).$$

2.
$$ye^{xy} + 2(x+y)^3 + 3x^4 + \sin y = C.$$

3.
$$y(t) = 2(t^2-1) + Ce^{-t^2}.$$

4.
$$y(x) = Ae^x + Bx.$$

5.
$$y(t) = 3e^t\cos 2t + 2e^t\sin 2t + 2\sin t.$$

6.
$$y(t) = e^{-9t}(c_1 + c_2t + 3t^2).$$

7.
$$y(t) = e^t[2t\arctan t - \ln(1+t^2) + c_1 + c_2t].$$

8.
$$y(x) = A(1-2x^2) + B\left[x + \sum_{n=1}^{\infty}\frac{(2n-3)(2n-5)\dots(-1)}{(2n)(2n-2)\dots(2)}x^{2n+1}\right].$$

9.
$$y(x) = Ax^{1/2}[1 + \sum_{n=1}^{\infty}\frac{(-1)^n x^{2n}}{n!(4+3)\dots(4n+3)}] + Bx^{-1}[1 + \sum_{n=1}^{\infty}\frac{(-1)^n x^{2n}}{n!(4-3)\dots(4n-3)}].$$

10.
$$e^{-5s}(\frac{2}{s^3} + \frac{8}{s^2} + \frac{16}{s}).$$

11.
$$F(s) = \frac{s^2(s^2-6s+8)}{(s^2-6s+10)^2} - 1.$$

12.
$$y(t) = [1 + u_{2\pi}(t)]\sin t.$$