MATH344 Calculus III Assessment

1. Evaluate the integral. (a)

(b) $\int \frac{1+x}{1+x^2} dx,$ $\int \sinh(x) \cosh(x) dx.$

2. Find the limit, if it exists. If it does not exist, explain why. (a) $\sinh(x) = x$

(b)
$$\lim_{x \to 0} \frac{\sin(x) - x}{x^3},$$
$$\lim_{x \to \infty} x \sin(1/x).$$

3. Evaluate the integral. (a)

$$\int e^{\theta} \sin(\theta) d\theta,$$
 (b)

$$\int \sin^2(x) \sin(2x) dx,$$

(c)
$$\int_0^{2/3} \sqrt{4 - 9x^2} dx,$$

(d)

$$\int_2^\infty \frac{dv}{v^2 + 2v - 3}.$$

4. Find the area of the surface obtained by rotating the curve about the x-axis.

$$y = x^3, \quad 0 \le x \le 2.$$

5. Find the area of the region inside the polar curve $r = 3\sin(\theta)$ and outside the polar curve $r = 1 + \sin(\theta)$.

6. Determine whether the series is absolutely convergent, conditionally convergent, or divergent, and prove it.

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+1}}.$$

7. Find the Taylor series expansion for $f(x) = \ln(x)$ about a = 2, and determine its interval of convergence.

8. Find the equation for the line through the point (-2, 2, 4) and perpendicular to the plane 2x - y + 5z = 12.

9. Find the equation for the plane through the three points P = (1, 2, 1), Q = (1, 0, 1), and R = (1, 1, 0).