Math 511

Q's 6.3 30a $A = \begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ $Y_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$Y' = AY, \quad Y(0) = Y_0$

Sol'n $\boxed{Y = e^{tA} Y_0}$ Sol'n

$$\boxed{Y = e^{t\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$$

(if) $A = XDX^{-1}$ (eigenvalue/vectors)

$$Y = X e^{tD} X^{-1} Y_0$$

$A = \begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ $Y_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$r_1 = 1$

$\begin{bmatrix} 0 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$x_1 = 2 \quad x_2 = 0$

$x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$r_2 = -1$

$\begin{bmatrix} 2 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$x_2 = 2 \quad x_1 = 2$

$x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1}$$

$$Y = X e^{tD} X^{-1} Y_0$$ ← $Y' = AY$, $Y(0) = Y_0$

$$Y = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e^t & 0 \\ 0 & e^{-t} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & | & 0 \\ 0 & 1 & | & 1 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e^t & 0 \\ 0 & e^{-t} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ e^{-t} \end{bmatrix} = \begin{bmatrix} e^{-t} \\ e^{-t} \end{bmatrix}$$

Final Review (+) Donuts ($11^{30} - 12^{20}$ Friday (here))

Exam 3 — 12 probs (10 pts each)

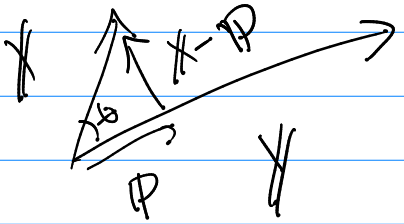
110 pts = 100%

5.1 – 5.6 orthogonality (8 probs)

6.1 – 6.3 Eigen value / vectors (4 probs)

5.1 $\mathbb{R}^n$, $X^T X$ (1 prob)

① $\mathbb{R}^n$ be able to do vector projections



$p \perp x-p$

for p (find scalar and vector proj.)

5.2 $R(A), R(A^T), N(A), N(A^T)$ (1 prob)

$\cos\theta = ?$

(orthogonal subspaces)

① given A q's: find these?
- use these?
- show $\perp$ for elements?

5.3 least squares prob (1 problem)

① Application: Data fit

Ex pts

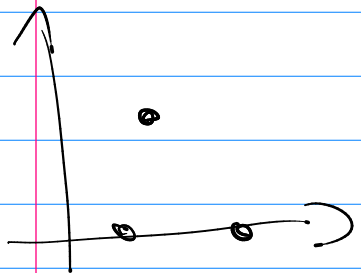
x	y
x_1	0
1	1
2	0

fit $y = ax + b$

$0 = a \cdot x_1 + b$

$1 = a \cdot 1 + b$

$0 = a \cdot 2 + b$



$$\begin{bmatrix} 1/2 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1/2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

2×3 $\quad$ 3×2

$$\begin{bmatrix} 2\times 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

5.4 Inner Prod. Spaces → ① $C[a,b]$, $\langle f, g \rangle =$ ☐

2 probs → ② $\mathbb{R}^{m \times n}$, $\langle A, B \rangle =$ ☐

- find projections
- find inner prod's
- find lengths
- find cos θ

5.5 Orthonormal 2 probs

① show orthonormal

② use orthonormal → coord. → find or use $c_1 = \langle f, g_1 \rangle$

hw g_1, g_2 are orthonormal $f = c_1 g_1 + c_2 g_2$ $c_2 = \langle f, g_2 \rangle$

use $f = c_1 g_1 + c_2 g_2 \qquad h = k_1 g_1 + k_2 g_2$

$\langle f, h \rangle = c_1 k_1 + c_2 k_2$

ex $\sin x, \cos x$ are orthonormal on

$C[-\pi, \pi] \quad \langle f, g \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f g \, dx$

$h_1(x) = 3\sin x + 2\cos x$

$h_2(x) = \pi \sin x - e \cos x$

$$\frac{1}{\pi} \int_{-\pi}^{\pi} (3\sin x + 2\cos x)(\pi \sin x - e\cos x)\, dx$$

$= 3\pi - 2e$

56 Gram-Schmidt 1 prob

① 2-vectors of $\mathbb{R}^3$, run gram-schmidt

$v_1, v_2 \to q_1, q_2$

61 2 probs Eigen Vector/Value

① $A =$ solve $\lambda_i =$

② $x_i =$

6.2 $Y' = AY$, $Y(0) = Y_0$ word problem

Sol: $Y = e^{tA} Y_0$ $\quad A = XDX^{-1}$

$\rightarrow$ $Y = X e^{tD} X^{-1} Y_0$

ex word problem: " blah blah blah ... " $\rightarrow$ $A = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$

$$A = XDX^{-1}, \quad Y_0 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}^{-1}$$

$$Y = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} e^t & 0 & 0 \\ 0 & e^{2t} & 0 \\ 0 & 0 & e^{-t} \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} e^t \\ e^{2t} \\ 0 \end{bmatrix} = \begin{bmatrix} e^t \\ e^{2t} \\ e^{2t} \end{bmatrix} = Y$$

Q3 $AX = XD \rightarrow A = XDX^{-1}$ (Prob)

or

$X^{-1}AX = D$

① Show D, X