Math 511

http://chaos.math.wichita.edu/

Ch1 this is not just Matrix Arithmetic.

Modeling

$$\begin{cases} 3a + 2c = 7 \\ a + 2c = 6 \end{cases}$$

Sys. of linear eqn's.

$a = 6 - 2c$

Solve

→ substitution ⇒ $a + 2c - 2c = 6 - 2c$

→ elimination

$$\Rightarrow \begin{cases} 3a + 2c = 7 \\ a + 2c = 6 \end{cases} \rightarrow \begin{array}{r} 3a + 2c = 7 \\ -3a - 6c = -18 \end{array}$$

$$-4c = -11$$

(you finish)

Terms

M eqn's, n-unknown

$$\begin{cases} c_{11}x_1 + c_{12}x_2 + \cdots + c_{1n}x_n = d_1 \\ c_{21}x_1 + c_{22}x_2 + \cdots + c_{2n}x_n = d_2 \\ \vdots \\ c_{m1}x_1 + c_{m2}x_2 + \cdots + c_{mn}x_n = d_m \end{cases}$$

(M x n) system

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \cdots & c_{1n} \\ c_{21} & c_{22} \cdots & c_{2n} \\ \vdots & & \\ c_{m1} & - & - \cdots & c_{mn} \end{bmatrix} \qquad \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_m \end{bmatrix}$$

$m \times n$ matrix $\qquad$ $m \times 1$ matrix

1) Augmented Matrix

$$\left[\begin{array}{ccc|c} c_{11} & \cdots & c_{1n} & d_1 \\ \vdots & \ddots & \vdots & \vdots \\ c_{m1} & \cdots & c_{mn} & d_m \end{array}\right]$$

How to Solve?

→ Elimination makes equivalent systems.
(substitution)

↓

Give same solns.

→ ops. that keep same solutions.

1. Switch rows
2. Mult. by non-zero
3. Mult. row + row = New row

1) Matrix ops.

1. Switch rows (interchange)
2. Mult. by non-zero
3. (const) row + row = New row
   ↑ non-zero

to solve quickly → $(-2)r_1 + r_2 = \text{New } r_2$

$r_3 - r_1 = \text{New } r_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 2 & 1 & 0 & -1 \\ 1 & 3 & 5 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & 1 & 2 & -3 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & -3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 6 \end{array}\right]$$

No Soln

$0 \cdot x + 0 \cdot y + 0 \cdot z = 6$

---

goal: strict triangular

$$\left[\begin{array}{ccccc|c} x & x & x & \cdots & x & \\ 0 & x & x & \cdots & x & \\ 0 & 0 & x & \cdots & x & \\ 0 & 0 & 0 & \cdots & x & \\ 0 & \vdots & 0 & 0 & x & \end{array}\right]$$