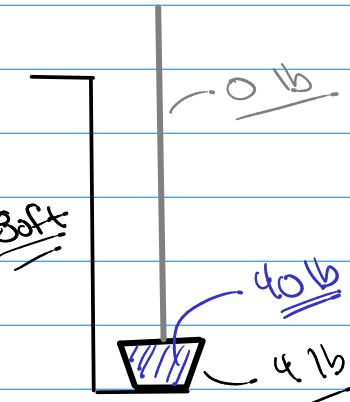
Math 242

Qs/ 5.4 #18



bucket is going up @ 2ft/sec
and leaks @ $\frac{1\text{ lb}}{5\text{ sec}} = .2$ lb/sec

Work to lift bucket 80ft
= ?

$$\text{Total Work} = \int_{\text{bottom of well}}^{\text{top of well}} (\text{Small work}) = \int_{\text{bott}}^{\text{top}} (\text{weight})(dx)$$

Small dep.

(ex) didn't leak → $\text{Work} = \int_0^{80} 44\,dx$

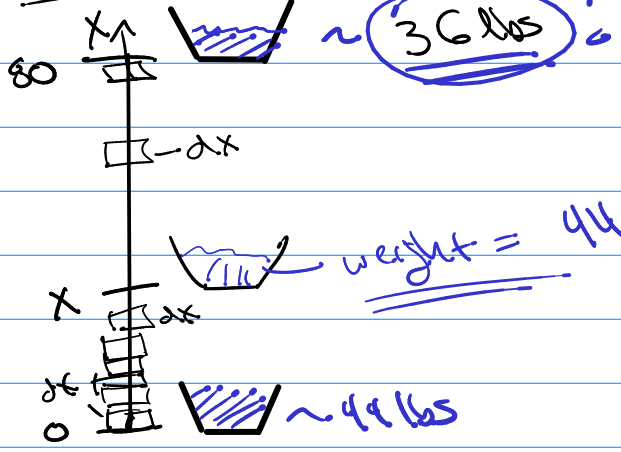
$= 44\left(x\Big|_{x=0}^{x=80}\right)$

$= 44 \cdot 80$

$= 3520$ ft·lb

---

but it's leaking



Moving @ 2ft/sec
leaking @ .2 lb/sec

$\frac{.2\text{ lb/sec}}{2\text{ ft/sec}} = .1$ lb/ft

$-\frac{1}{10}x$ vs 10ft/lb

$$\int_0^{80} \left(44 - \tfrac{1}{10}x\right) dx$$

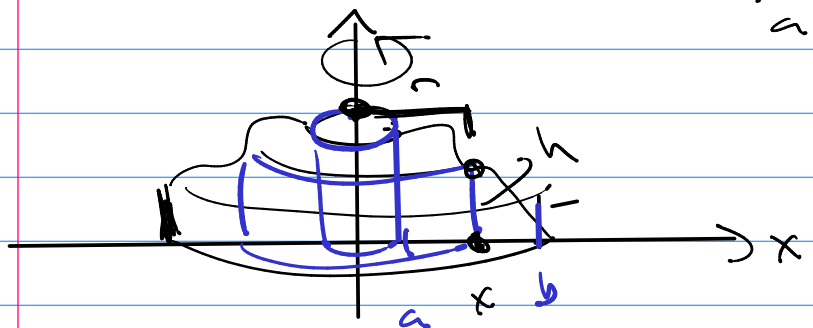
$$W = 44x - \frac{1}{20}x^2 \Big|_0^{80} = \text{finish}$$

5.3 #29 — Shape → setup → $V = \int_0^3 2\pi x^5\,dx$

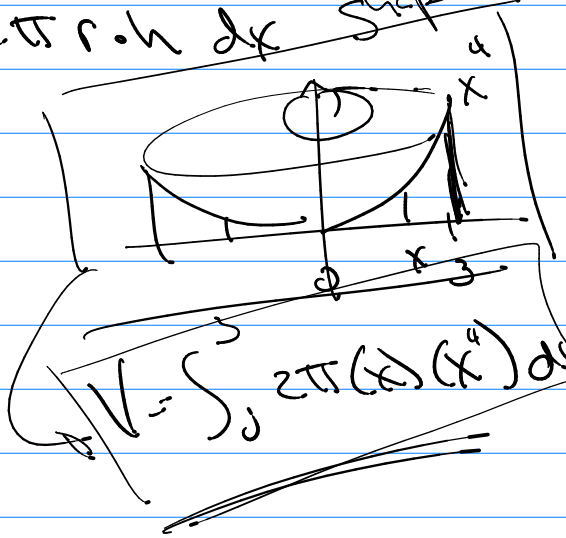
↳ section is shells

$2\pi rh$

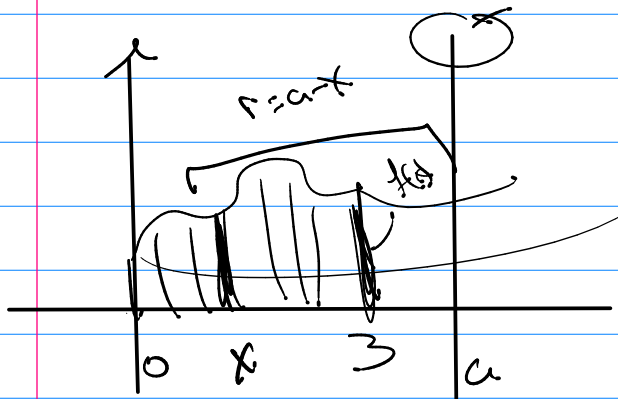
$$V = \int_a^b 2\pi r \cdot h\,dx$$



Shape



$$V = \int_0^3 2\pi(x)(x^4)\,dx$$



$$\int_0^3 2\pi\,(a-x)\,f(x)\,dx$$

x^5

Exam 4 — 21 probs @ 10 pts each

200 pts = 100%

4.1/4.2 — 2 probs

① Area by limit — ex $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$

$x_i = a + i\,\Delta x$ — right endpts

$\Delta x = \frac{b-a}{n}$

Ex $\int_0^3 x^2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(0 + i\frac{3}{n}\right)^2 \left(\frac{3}{n}\right)$

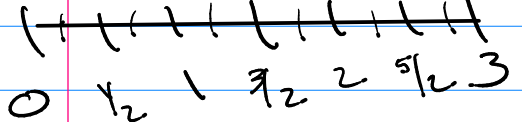
$\Delta x = \frac{3-0}{n}$ $= \lim_{n\to\infty} \frac{27}{n^3} \sum_{i=1}^{n} i^2$

$= \lim_{n\to\infty} \frac{27}{n^3} \frac{n(n+1)(2n+1)}{6} = \frac{9}{2} \lim_{n\to\infty} \frac{n(n+1)(2n+1)}{n^3}$

$= \frac{9}{2} \lim_{n\to\infty} \frac{2n^3}{n^3} = \boxed{9}$

② Approximate (set up)

Ex $\int_0^3 x^2\,dx$ using 6 intervals and mid pt. approx.



$\Delta x = \frac{3-0}{6} = \frac{1}{2}$

mid pts: $1/4, 3/4, 5/4, 7/4, 9/4, 11/4$

$\int_0^3 x^2\,dx \approx \left(\left(\frac{1}{4}\right)^2 + \left(\frac{3}{4}\right)^2 + \left(\frac{5}{4}\right)^2 + \left(\frac{7}{4}\right)^2 + \left(\frac{9}{4}\right)^2 + \left(\frac{11}{4}\right)^2 \right)\left(\frac{1}{2}\right)$

Stop here

4.3/4.5 10 probs

1-5 $\int f(x)\,dx = F(x) + C$

Ex $\int (\sin x + \sec x\tan x + x^3 + 3)\,dx$

Hwd ish $\int x\sqrt{x+3}\,dx$

ex $\int x\sqrt{2x+3}\,dx = \int \frac{u-3}{2}\sqrt{u}\,\frac{1}{2}du$

$u = 2x+3$

$du = 2dx$

$= \frac{1}{4}\int (u-3)u^{1/2}\,du$

$= \frac{1}{4}\int (u^{3/2} - 3u^{1/2})\,du$

$= \frac{1}{4}\left[\frac{2}{5}u^{5/2} - 3\cdot\frac{2}{3}u^{3/2}\right] + C = \frac{1}{10}(2x+3)^{5/2} - \frac{1}{2}(2x+3)^{3/2} + C$

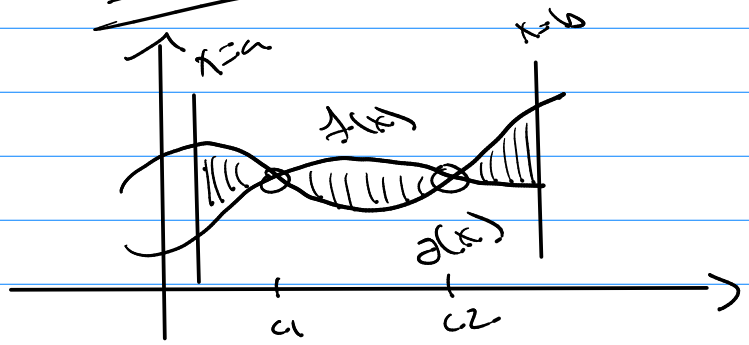
6-10 Similar to 1-5 except $\int_a^b f(x)\,dx$

$= F(b) - F(a)$

Note: if you get $\sin(3) - \sin(2\sqrt{7})$ Stop

5.1 Area between curves 2 probs

① Setup only



a) picture

b) intersections

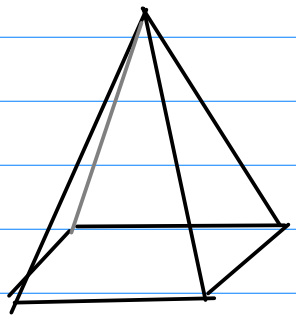
c) Area = ?

↑ def. integrals

(2) Setup and Solve

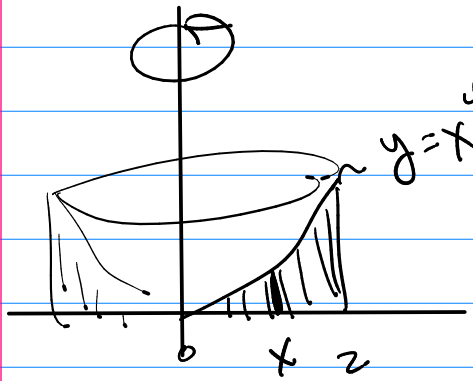
5.1-5.3 (3 probs)

(1) Volume of cone (circular or square or rectangle)

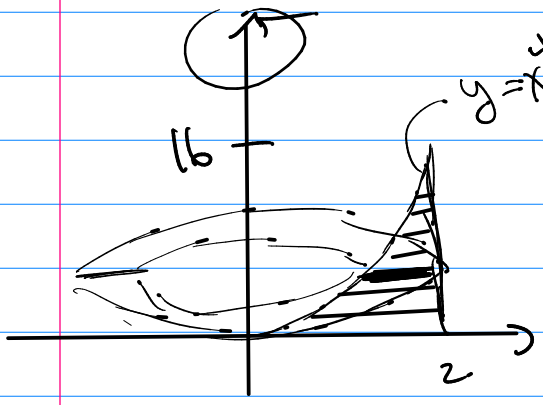


(2) Volume by disks/washers.

(3) Volume by shells.



shells: $\int_0^2 2\pi x \cdot x^4 \, dx = \int_0^2 2\pi x^5 \, dx$



disk/washers

$$V = \int_{y=0}^{y=16} \left(\pi(2)^2 - \pi\left(y^{1/4}\right)^2\right) dy$$

$$= \pi \int_0^{16} \left(4 - y^{1/2}\right) dy =$$

5.4 Work (2 probs)

(1) Spring

(2) empty a water tank (Setup)

5.5 $f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$ (2 probs)

(1) find f_{ave}

(2) find $x=c$ where $f(c) = f_{ave}$

Ch 6 Inverse functions for composition.

Math = toys + rules

objects operations

Undo an operation ex:

$3x + 7 = 1$

$3x + 7 + (-7) = 1 + (-7)$ (-7 is +7's additive inv.)

$3x + 0 = -6$ (0 is addition identity)

Composition of functions

idea: $\sin(3x+2) = 1$ Solve?

$3x+2$ = something

---

(1) Notation: $(f \circ g)(x) = f(g(x))$

(2) Identity & composition of functions: $I(x)$

want: $f(I(x)) = f(x)$

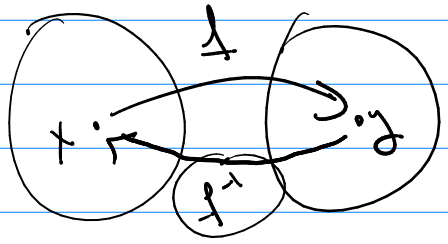
$I(f(x)) = f(x)$

let $I(x) = x$, $I(\square) = \square$

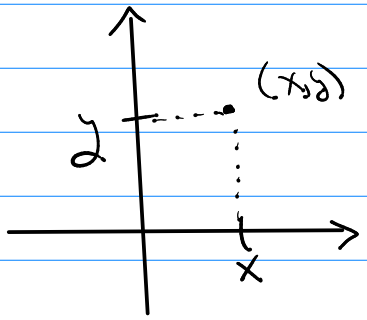
check: $I(x^3) = x^3$ $\quad (I \circ f)(x) = (f \circ I)(x)$

$(I(x))^3 = x^3$ $\quad = f(x)$

Can $f(x)$ have an inverse?



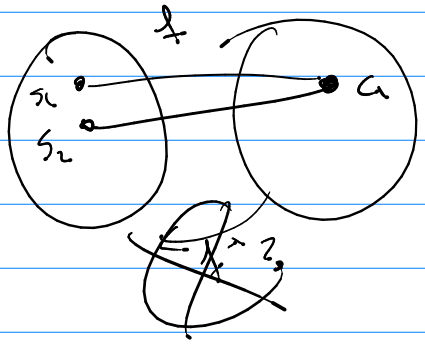
$y = f(x)$

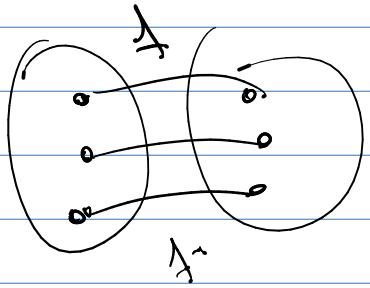


f^{-1} is inverse function

when can f^{-1} exist?

f is one-to-one





f^{-1} exists if f is one-to-one (passes horz. line test)

$f(x) = x^3$ has an inverse $f^{-1}(x) = ?$