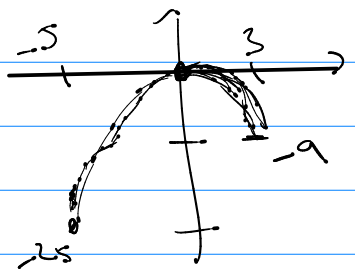
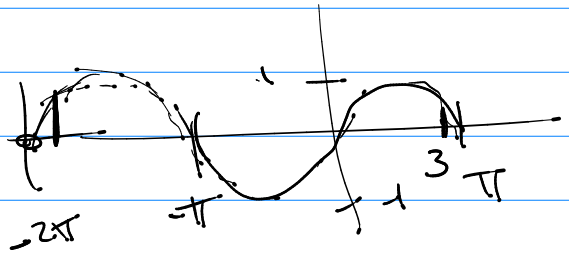
Math 242

3.6 Graphing → Calculus
→ Computers

$f(x) = \sin(x) - x^2 \qquad -5 \le x \le 3$





$f'(x) = \cos x - 2x$
$f''(x) = -\sin x - 2$

Criticals $f'(x) = 0$, $f'(x)$ dne

$\cos x - 2x = 0$ never

$\cos x = 2x$

$x \approx 0.45$

f' + | 0.45 | −

rel. max → abs. max

location & max

(0.45, 0.9)

$f''(x) = -\sin x - 2$

possible inflection $-\sin x - 2 = 0$

$-\sin x = 2$ never

f'' − | 0

$$f(x) = \sin(ax) - x^2$$

$$f'(x) = a\cos(ax) - 2x$$

$$a\cos(ax) - 2x = 0$$

$$\cos(ax) = \frac{2}{a}x$$

$$f''(x) = -a^2\sin(ax) - 2$$

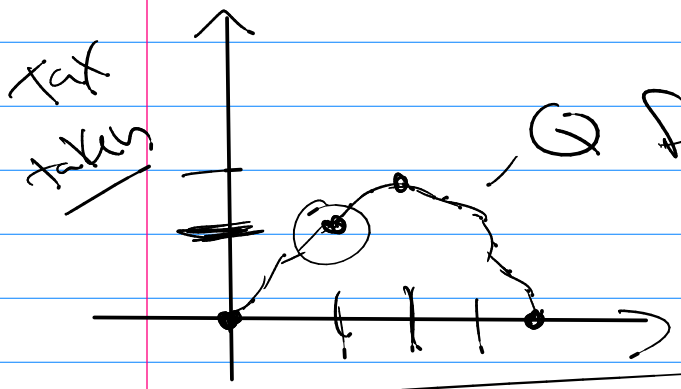
$$-a^2\sin(ax) - 2 = 0$$

$$\sin(ax) = \frac{-2}{a^2}$$

$a \geq \sqrt{2}$!

3.7 Applications (Optimization Problems)

Find max and/or min (absolute)



Q Find extreme values of our (gravity.)

Solving Optimization Problems

Polya's Method (any problem)

0) What are your abilities?
1) Understand problem
2) Make a plan
3) Carry out the plan
4) check

Optimization

1. Understand the problem. (Read)
2. Draw!
3. Symbols, equations, functions (pick useful labels)
4. What quantity are you optimizing?
   → Find its function
5. Make it a function of one variable. (Domain?)
6. Find extrema

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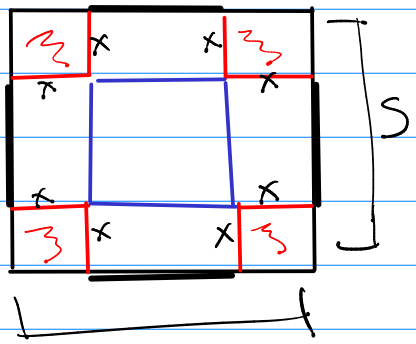
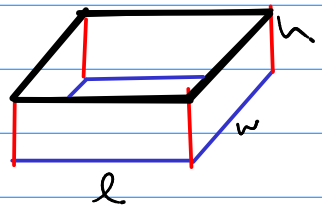
ex



$0 \le x \le S/2$



$h = x$

$l = S - 2x$

$w = S - 2x$

$V = l \cdot w \cdot h$

$V = (S-2x)^2 x$

S is a constant.

$$V(x) = x(S-2x)^2 \quad 0 \le x \le S/2, \ S \text{ is a constant}$$

Yes, this has an abs. max and abs. min.

Where is abs. max? check end points + criticals

$$V'(x) = (1)(S-2x)^2 + (x)(2)(S-2x)(-2)$$

$$V'(x) = (S-2x)\left[(S-2x) - 4x\right]$$

$$V'(x) = (5-2x)(5-6x)$$

Criticals. $V'(x)=0$ $V'(x)$ dne never

$5-2x=0$ $5-6x=0$

$x = 5/2$ $x = 5/6$

$V'(x)$: $+$ on $(0, 5/6)$, $-$ on $(5/6, 5/2)$

rel. max only one → abs. max here.

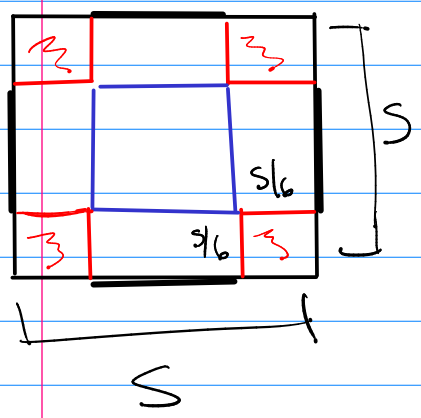
Abs Max @ $x = 5/6$ & $V = \frac{2}{27}s^3$

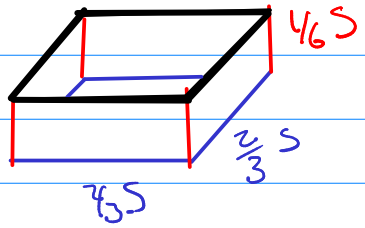
$V = (5-2x)^2 x$

$V = (5 - \frac{5}{3})^2 (5/6)$

$V = (\frac{2}{3}5)^2 (5/6)$

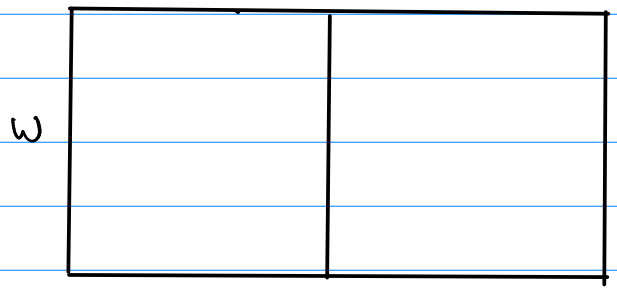
$V = \frac{2}{27}s^3$





Ex Area of a rectangular field = 1.5 million ft²

Minimize Cost of fence



Area = $l \cdot w$

fence = $l + l + w + w + w$

Cost = (fixed price)(fence)

$1.5\ \text{Mft}^2 = l \cdot w \rightarrow l = \frac{1.5\ \text{Mft}}{w}$

$C = c(2l + 3w)$

$C(w) = c\left(\frac{3\ \text{Mft}^2}{w} + 3w\right)$

Minimize: $C(w) = c\left(\frac{3 \times 10^6}{w} + 3w\right)$

$0 < w < +\infty$ Domain: $(0, +\infty)$