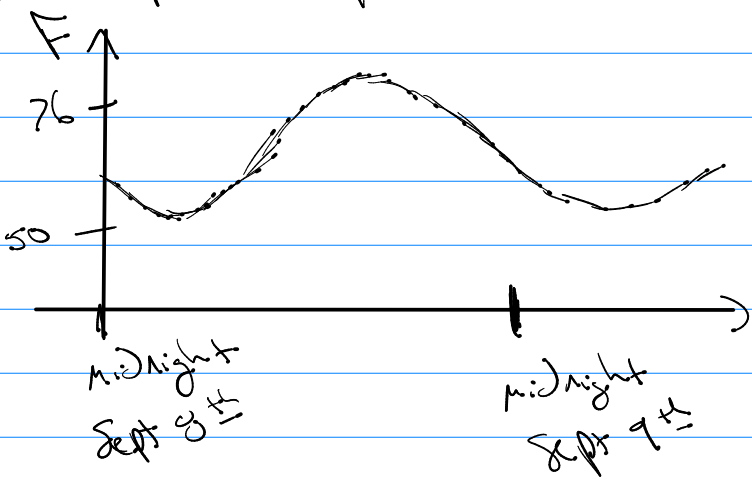
Math 242

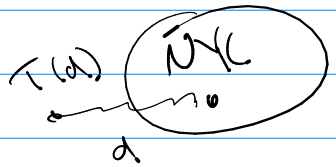
Q5/ 1.8 (10)

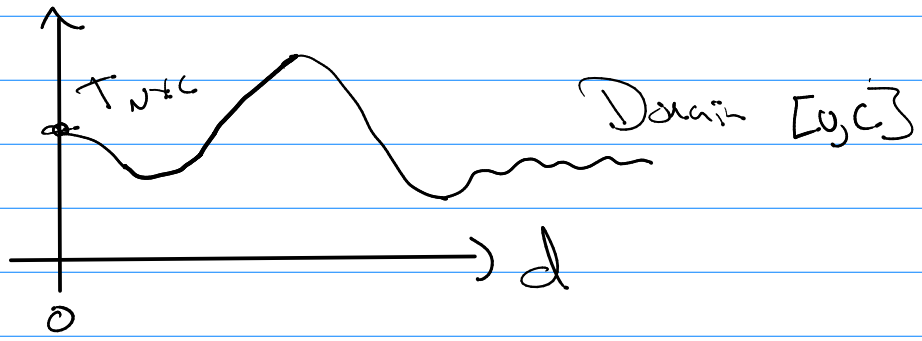
(0a) temp @ a place as a function of time



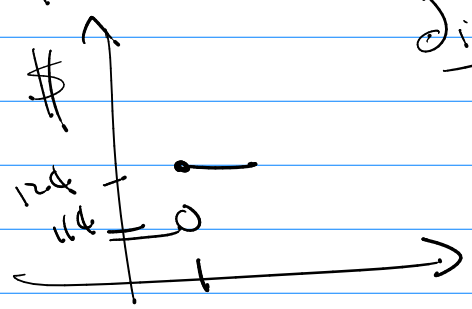
- cont.
- Domain is $\mathbb{R}$
- Codomain is $\mathbb{R}$
- Range [low temp, high temp]

(0b)





(0c) # Sar Mils

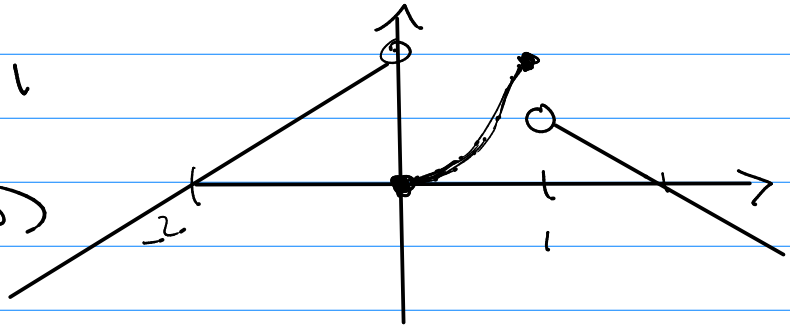


disc.

$$f(x) = \begin{cases} x+2 & x<0 \\ 2x^2 & 0 \le x \le 1 \\ 2-x & x>1 \end{cases}$$

Cont: $(-\infty,0) \cup (0,1) \cup (1,\infty)$

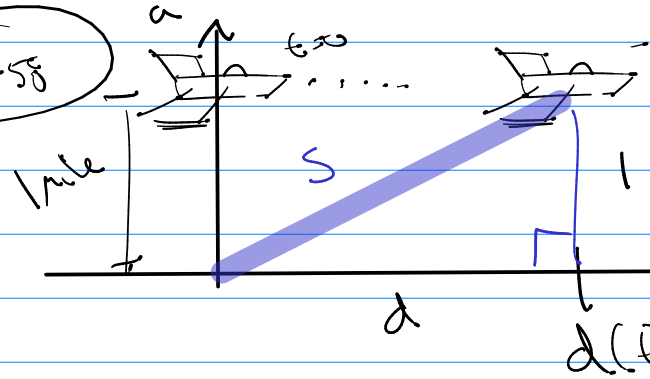
Disc: $0, 1$



$2x^2$ is cont from the right @ $x=0$

$2x^-$ is cont from the left @ $x=1$

1.3 #58



speed = 350 mi/h $\quad d = rt$

$d^2 + 1^2 = s^2$

$s = (1+d^2)^{1/2}$

$s(d) = (1+d^2)^{1/2}$ $\quad$ Note: d, s are in miles, t is in hours

$d(t) = 350\,t$

$s(t) = s(d(t)) = (1 + (350t)^2)^{1/2}$

Exam 1 $\quad$ 80 min $\rightarrow$ 21 probs @ 10 pts each, 200 pts = 100%

1.1 Function Basics (2 probs)

① Word problem $\rightarrow$ sketch a rough graph.
Domain, Codomain, range

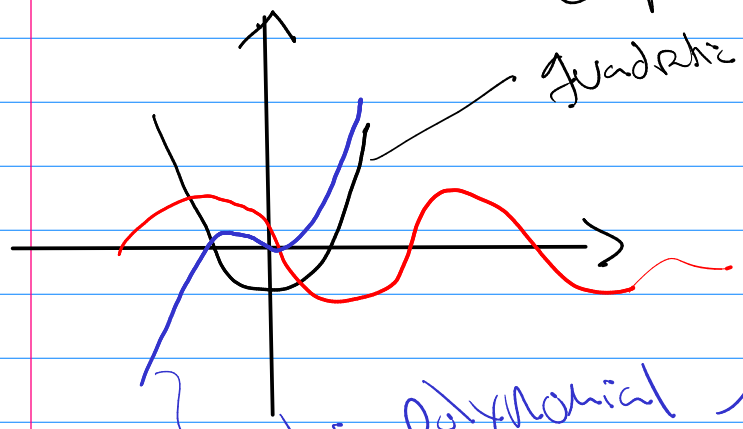
② $f(-x) \rightarrow -f(x)$ odd functions

$\rightarrow f(x)$ even functions

1.2 Functions you are supposed to Know (2 pobs)

① ② - Name functions (ex. Name → example w/ graph ←)

- Match graphs



quadratic polynomial $f(x) = ax^2+bx+c$

$f(x) = \sin(x)$

cubic polynomial $f(x) = ax^3+bx^2+cx+d$

1.3 ops on $f(x)$ (3 pobs)

① translations / stretchings

ex. know graph of $y = \sin x$

rough graph $y = 3\sin(\pi(x-1)) + 2$

② ops: $f+g$, $f-g$, fg, f/g, $f \circ g$

③ Like 1.3 #55 to #58

1.4 (0 probs)

1.5 Intuitive Limits (3 probs)

①,② Limits by graphs / tables $\lim_{x \to a} f(x) = L$

③ Infinite Limits $\lim_{x \to a} f(x) = \begin{matrix} +\infty \\ -\infty \end{matrix}$

1.6 Limit Laws (4 probs)

①-③ Use limit laws (or not!)

$$\lim_{x \to a} f(x) + g(x) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

etc.

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \quad \text{as long as} \quad \lim_{x \to a} g(x) \neq 0$$

$$\lim_{x \to a} \frac{x+a}{x-a} = \frac{\lim_{x \to a} (x+a)}{\lim_{x \to a} (x-a)} =$$

④ etc. $\lim_{x \to 1} \dfrac{\sqrt{x} + x^3}{x + \sqrt{x}} = \dfrac{\sqrt{1} + (1)^3}{1 + \sqrt{1}} = 1$

④ Use the squeeze thm.

Show

Ex $\lim_{x \to 0} x^4 \cos\left(\frac{2}{x}\right) = 0$

1.7 ε-δ proofs (3 probs)

(1) limit of a linear function

(2) limit of a linear function

(3) limit of a quadratic function

Ex (Linear) $\lim_{x \to 2} 3x - 5 = 1$

$\forall \varepsilon > 0 \; \exists \delta > 0$ (if $0 < |x-2| < \delta$, then $|(3x-5) - 1| < \varepsilon$)

Scratch $|(3x-5)-1| < \varepsilon \to |3x-6| < \varepsilon$

$\to |3(x-2)| < \varepsilon \to 3|x-2| < \varepsilon$

$\to |x-2| < \varepsilon/3$

Pf Let $\delta = \varepsilon/3$

$0 < |x-2| < \delta \to |x-2| < \varepsilon/3 \to 3|x-2| < \varepsilon$

$\to |3x-6| < \varepsilon \to |(3x-5)-1| < \varepsilon$ ∎

(ex) $\lim_{x \to 1} 1 - 3x = -2$

Show: $\forall \varepsilon > 0 \; \exists \delta > 0$ (if $0 < |x-1| < \delta$, then $|(1-3x)-(-2)| < \varepsilon$)

Scratch: $|(1-3x)-(-2)| < \varepsilon \rightarrow |3-3x| < \varepsilon$

$\rightarrow 3|1-x| < \varepsilon$ etc...

1.8 (3 probs) Continuity

Is it cont'd

(1) Where is the given $f(x)$ continuous?

(ex) $f(x) = \dfrac{3x^2-1}{x+2}$ yes, on its domain all Reals but $x \neq -2$

(etc)

(2) Use continuity (limits)

(3) Intermediate Value th^{m} application

(ex) Does any number exist such that

(i) is one more than its cube?

$x = 1 + x^3 \rightarrow x^3 - x + 1 = 0$

$f(x)$

etc.