Math 242

Q5/ Functions to know (Algebra + graph + natural domain + codomain + range)

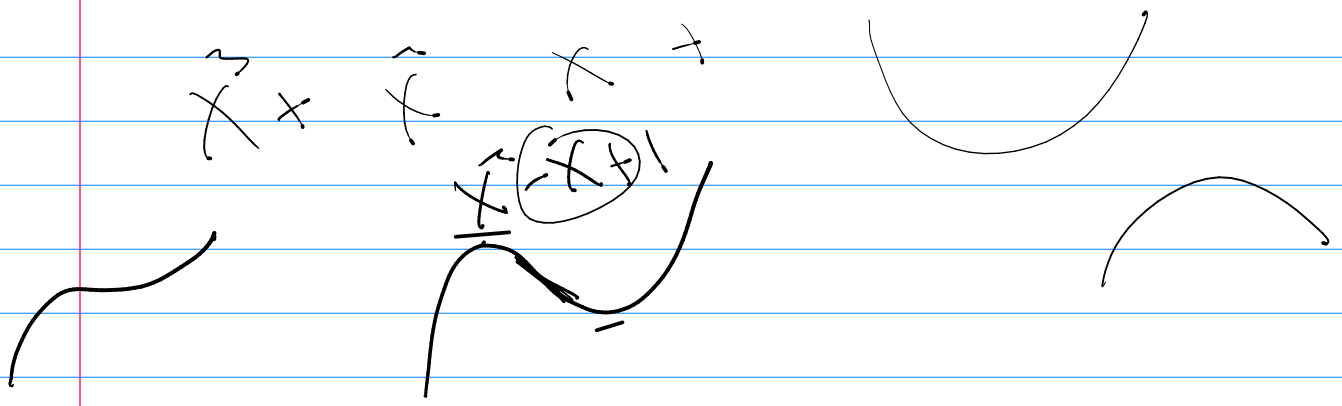
① $|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases} = \sqrt{x^2}$

② $p(x)$ is a polynomial

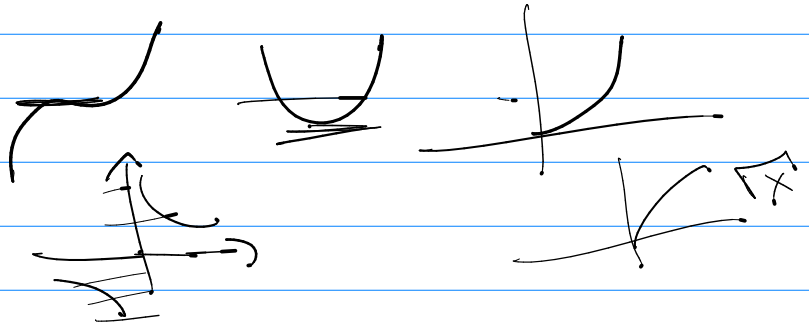
$p(x) = a_n x^n + \cdots + a_1 x + a_0$

odd.

even



③ $f(x) = x^n$ $\quad n = 0, 1, 2, \ldots$



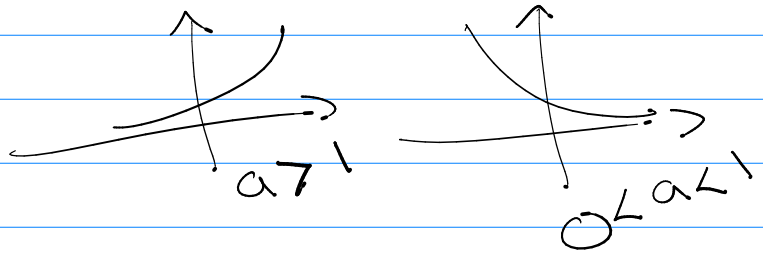
$n = -1 \quad \frac{1}{x}$

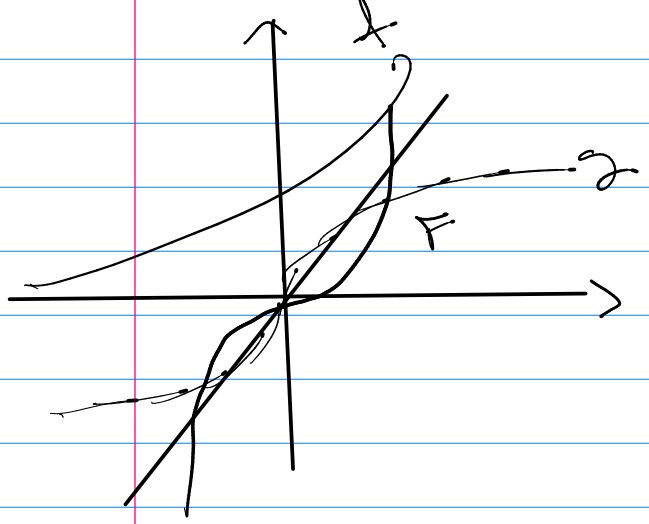
$n = \frac{1}{2}, \frac{2}{3}, \frac{1}{4}, \ldots$

④ $\frac{p(x)}{q(x)}$ ← $q(x) \neq 0$ $\quad \frac{(x+2)}{(x+2)(x-2)} = \boxed{\frac{1}{x-2}, x \neq -2}$

⑤ $\sin x$, $\cos x$, $\sec(x)$ $\csc(x)$ $\tan x$, $\cot x$

⑥ a^x





$y = 3x$ ⑥

$y = 3^x$ ①

$y = x^3$ ④

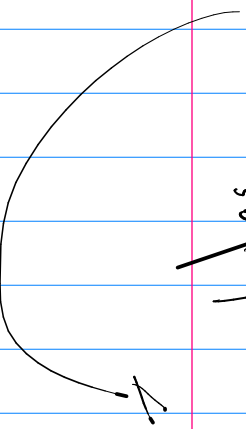
$y = x^{1/3} = \sqrt[3]{x}$ ⑤

Modeling: Cost (Miles Driven)

Mark	Cost = c	Dist. = d
1	$380	480 mi
2	$460	800 mi

linear model

$C(d) = s \cdot d + y_0$

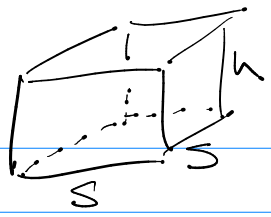


$s = \frac{y_2 - y_1}{x_2 - x_1}$

$y - y_1 = s(x - x_1)$

$s = \frac{460 - 380}{800 - 480} = \frac{8}{32} = \frac{1}{4}$ ¢/mi

1.1 #61



$V = s^2 h = 3$ m^3 → $h = 3/s^2$

$SA = 4sh + s^2$

$SA(s) = 4s\left(\frac{3}{s^2}\right) + s^2 = \frac{12}{s} + s^2$

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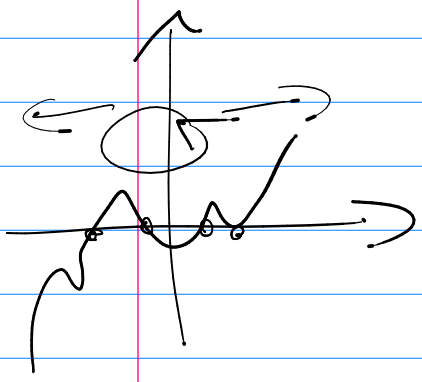
(13) Operations on functions

Part 1) "Same" function ... except we translate and/or stretch it

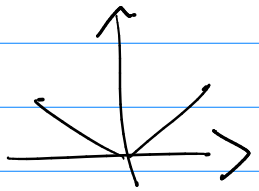
$f(x)$ (vs) $a \cdot f(b(x+c)) + d$

a, b, c, d are constants

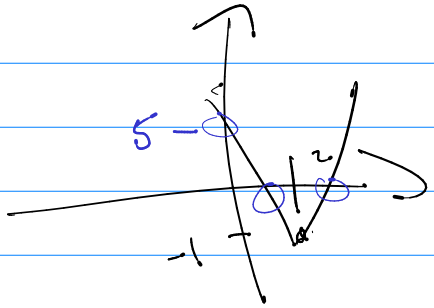


① $a f(x)$] stretch
② $f(bx)$]
③ $f(x) + d$] translation
④ $f(x+c)$]

(vs) $|x|$



(vs) $3|x-2| - 1$



Part 2 operations (+, −, *, ÷, composition)

$(f+g)(x) = f(x) + g(x)$

$(f-g)(x) = f(x) - g(x)$

$(f \cdot g)(x) = f(x)\,g(x)$

$(f/g)(x) = f(x)/g(x)$

$(f \circ g)(x) = f(g(x))$

Identity and Inverse for an operation

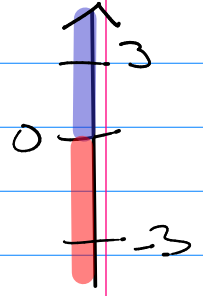
(ex) arithmetic: object = number
operator = addition

① Identity: does nothing to objects under operation

↳ additive identity $3 + \boxed{0} = 3$

② Inverse: make an object into the identity.

↳ additive inverse of 3 $3 + (-3) = 0$



$3x + 7 = 4$ $\quad x = 2$

$3x + 7 + (-7) = 4 + (-7)$

$3x + 0 = -3$

$\left(\frac{1}{3}\right)3x = \left(\frac{1}{3}\right)(-3)$

$x = -1$

$\sin(x)$ (b) inverse $\sin(x)$?

Under composition

Identity function $I(x) = x$

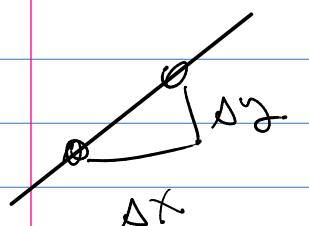
Inverse function $(f \circ f^{-1})(x) = x$

$(f^{-1} \circ f)(x) = x$

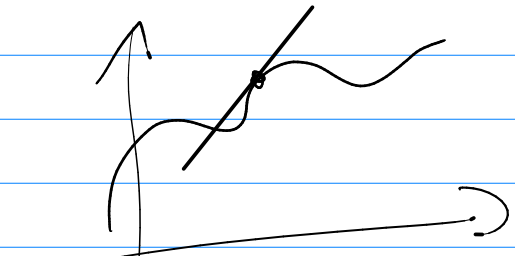
Calculus?

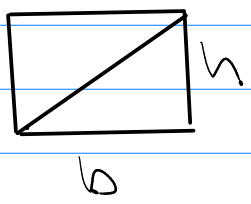
two problems: (1) change? (slope)

(2) sum? (area)



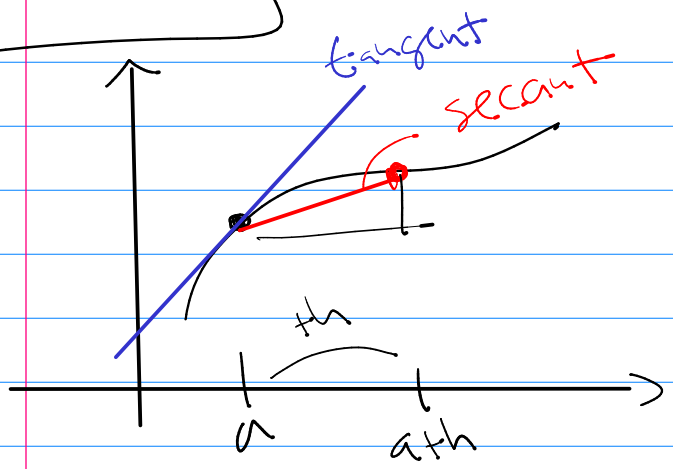
$m = \frac{\Delta y}{\Delta x}$





limit

Star change



slope of secant $= \frac{f(a+h) - f(a)}{h}$

let h get close to 0.

slope of secant → slope of tangent