Math 344

Exam 12 probs @ 10pts

16.1 (0 probs)

16.2 $\int_C 1\,ds$ (2 probs)

(1) $\int_C 1\,ds$ (ex) C is $r(t) = \langle t^3, t^4 \rangle$, $r' = \langle 3t^2, 4t^3 \rangle$, $1 \le t \le 2$

1 is $(x+y)$ or 1 is (x/y)

$$\int_a^b f(r(t))\,|r'(t)|\,dt$$

(ex) $$\int_1^2 (t^3 + t^4)\sqrt{(3t^2)^2 + (4t^3)^2}\,dt = ???$$

$1 = (x/y)$ or

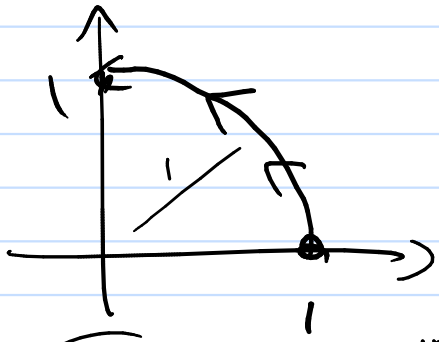
$$\int_1^2 \frac{t^3}{t^4}\sqrt{(3t^2)^2 + (4t^3)^2}\,dt = ???$$

(AS?) by hand?

(2) $$\int_C F \cdot dr = \int_C P\,dx + Q\,dy + R\,dz$$

$$= \int_a^b F(r(t)) \cdot r'(t)\,dt$$

(ex)



$x = \cos t$

$y = \sin t$

$0 \le t \le \pi/2$

say $F = \langle x^2, y^2 \rangle$

$$\int_C F \cdot dr = \int_0^{\pi/2} \langle \cos^2 t, \sin^2 t \rangle \cdot \langle -\sin t, \cos t \rangle \, dt$$

$F(r(t))$ $\qquad r'$

$$= \int_0^{\pi/2} \left(-\sin t \cos^2 t + \cos t \sin^2 t \right) dt = ???$$

16.3 Fund. th^n: $\int_C (\nabla f) \cdot dr = f(r(b)) - f(r(a))$

1 prob (a) show $F = \nabla f$

How?. $F = \langle P, Q \rangle$

show $P_y = Q_x$

(b) if $F = \nabla f$

$f_x = P$

$f_y = Q$ $\rightsquigarrow$ $f = ???$

(c) use 1

$$\int_C (\nabla f) \cdot dr = f(r(b)) - f(r(a))$$

(ex) $F = \langle e^{xy} + xye^{xy},\ x^2 e^{xy} \rangle$

C : $r(t) = \langle \cos t,\ 2\sin t \rangle$

$0 \le t \le \pi/2$ (a to b)

$\int_C F \cdot dr$?

$f = xe^{xy}$

$f_x = e^{xy} + xy e^{xy}$

$f_y = x^2 e^{xy}$

(a) check if $F = \langle P, Q \rangle$ $P_y = Q_x$ (do this)

(b) $f_x = e^{xy} + xy e^{xy}$, $f_y = x^2 e^{xy}$ $\longrightarrow$ $f =$ (do this)

(c) $\int_C F \cdot dr = f(r(b)) - f(r(a))$

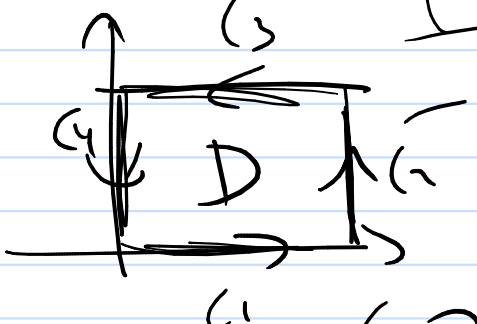
$(0,2)$ $\quad$ $(1,0)$

(do this)

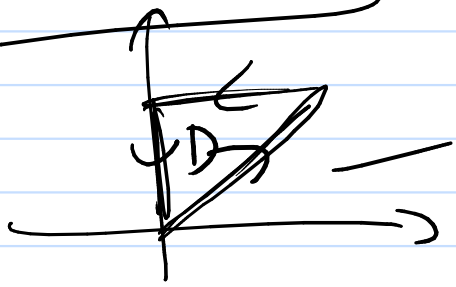
16.4 Green's Th^n (2 pts)

(1) $\oint_C F \cdot dr = \iint_D (Q_x - P_y)\, dA$

(ex)



(ex)



(ex)

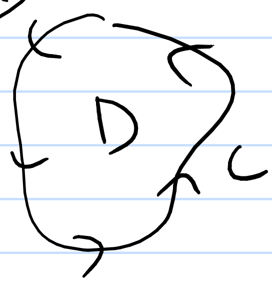
② $\text{area}(D) = \iint_D (1)\, dA = \oint_C x\, dy$ (dy → $y'\,dt$)

(1 ← $Q_x - P_y$)

↑ Find area of D → use!

or $-\oint_C y\, dx$

(ex)

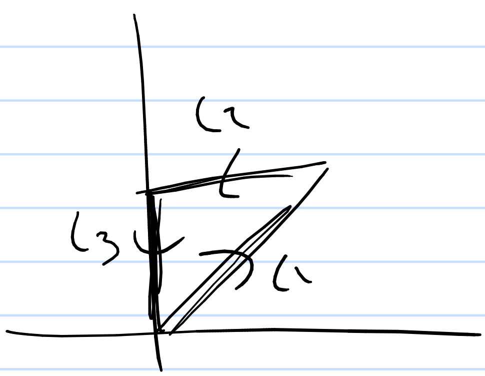


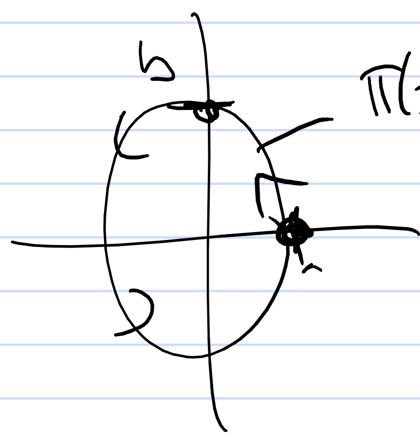
C is $\vec{r}(t) = \langle x(t), y(t) \rangle \quad a \le t \le b$

$\text{area}(D) = \oint_C x\, dy$

todo?

$= \int_a^b x(t)\, y'(t)\, dt$





$\vec{r}(t) = \langle a\cos t, b\sin t \rangle$

$0 \le t \le 2\pi$

16.5 Curl ($\mathbb{F}$) div ($\mathbb{F}$) (2 probs)

① Curl ($\mathbb{F}$) $\quad \mathbb{F} = \langle xy, z^2 \sin x, \sqrt{zy} \rangle$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & z^2 \sin x & \sqrt{zy} \end{vmatrix} = ???$$

② div ($\mathbb{F}$)

$= \nabla \cdot \mathbb{F}$

$\nabla \cdot \langle yz, x^3 \tan^4(xz), \sqrt{xy^3 + \sin x} \rangle = 0$

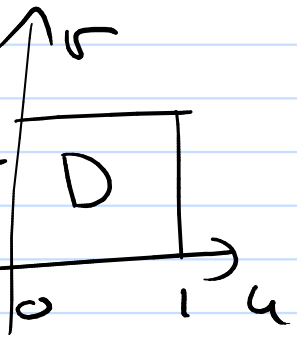
16.6 Parametrize Surface (0 probs)

16.7 Surface Integrals (3 probs)

① $\iint_S f\,dS = \iint_D f(\pi(u,v))\,|\pi_u \times \pi_v|\,dA$

(x) $\pi(u,v) = \langle u\cos v, u\sin v, u\rangle$

$f = xyz$



$\iint_S f\,dS = \iint_D (u^3\cos v\sin v)\,|\pi_u \times \pi_v|\,dA$

Note: $\pi_u \times \pi_v$

= ???

② Center of Mass (?!) like #34 p.1174

Setup $M = \iint_S \rho \, dS =$

$\bar{x} = \frac{1}{M} \iint_S x \rho \, dS =$

$\bar{y} = \frac{1}{M} \iint_S y \rho \, dS =$

$\bar{z} = \frac{1}{M} \iint_S z \rho \, dS =$

③ Flux (Heat Flow)

$\iint_S (F \cdot n) \, dS$ like example 6 p. 1172

(Note: u = temp @ (x,y,z))

$\iint_S F \cdot dS$

$\iint_D F \cdot (r_u \times r_v) \, dA$

16.8 Stokes (1 prob)

$\oint_C F \cdot dr = \iint_S [\text{curl } F] \cdot dS$

See probs 7, 9 pg 1179

(ex) Note: $r(u,v)$ is for $z = g(x,y)$

$\rightarrow \langle u, v, g(u,v) \rangle$

$|r_u \times r_v| = \langle -g_x, -g_y, 1 \rangle$ (x, y, z)

Thm: $\iint_D F \circ (r_u \times r_v)\, dA = \iint_D (-P g_x - Q g_y + R)\, dA$

16.9 Div. Thm (pob)

(1) $\iint_S F \cdot dS = \iiint_E \operatorname{div}(F)\, dV$

Given F, S in parametric form

Find $\iiint_E \operatorname{div}(F)\, dV$

Example 1, 2 p. 1163