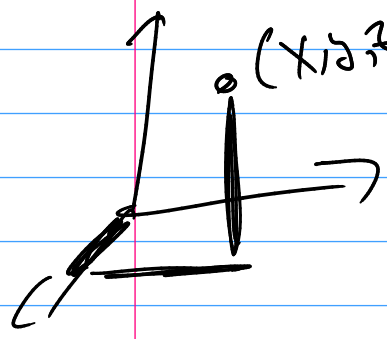
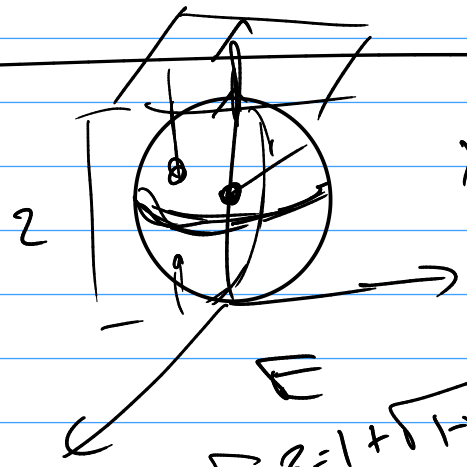
Math 314

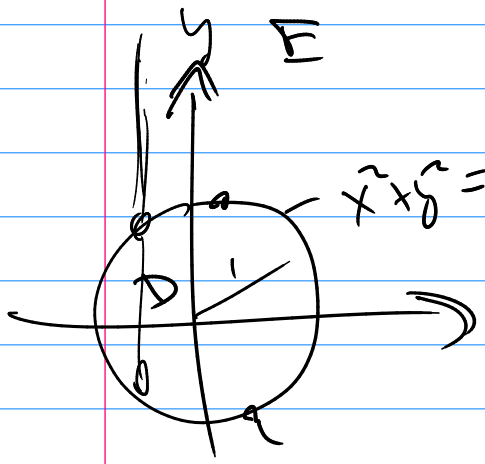
$(x-a)^2+(y-b)^2+(z-c)^2=r^2$



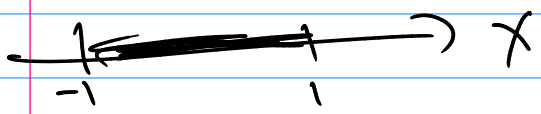


$x^2+y^2+(z-1)^2=1$

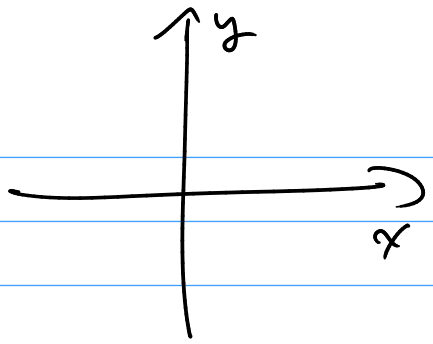
$$\iiint_E 1\,dV = \iint_D \left[\int_{z=1-\sqrt{1-x^2-y^2}}^{z=1+\sqrt{1-x^2-y^2}} 1\,dz\right] dx$$

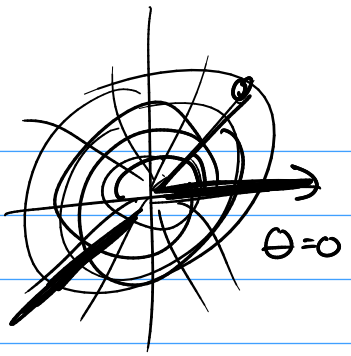


$$\int_{x=-1}^{1}\left[\int_{y=-\sqrt{1-x^2}}^{y=\sqrt{1-x^2}}\left[\int_{z=1-\sqrt{1-x^2-y^2}}^{z=1+\sqrt{1-x^2-y^2}} 1\,dz\right]dy\right]dx$$

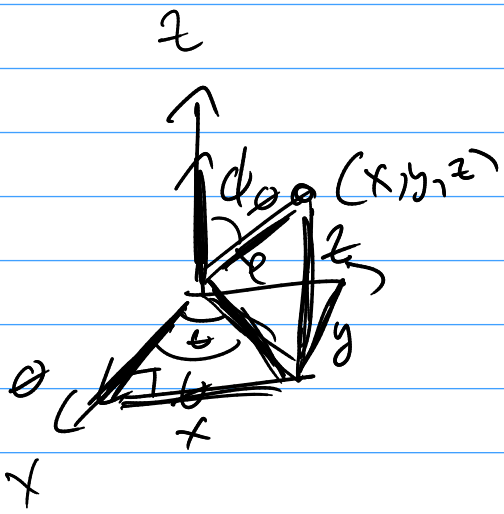


~~Spherical~~ Polar





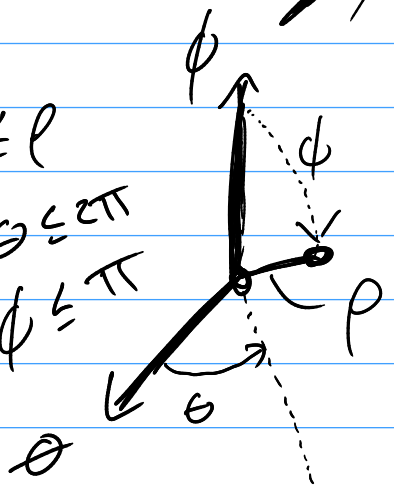
Spherical



$0 \le \rho$

$0 \le \theta \le 2\pi$

$0 \le \phi \le \pi$

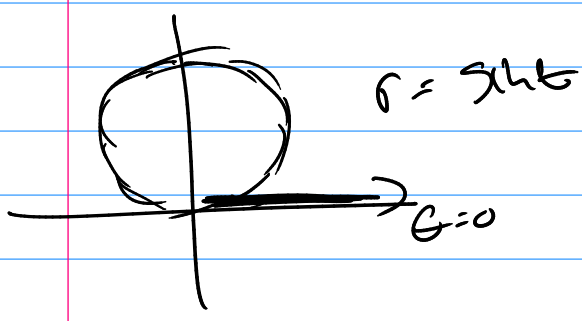


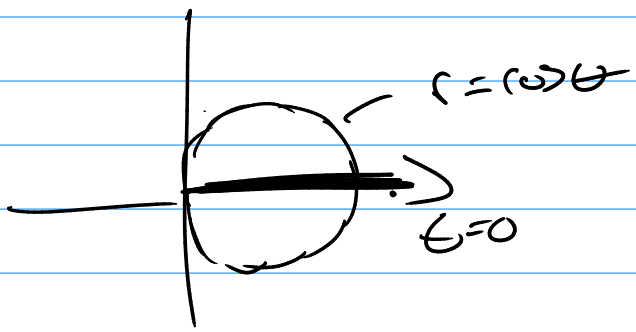
$$x = \rho \sin\phi \cos\theta \qquad x^2+y^2+z^2 = \rho^2$$

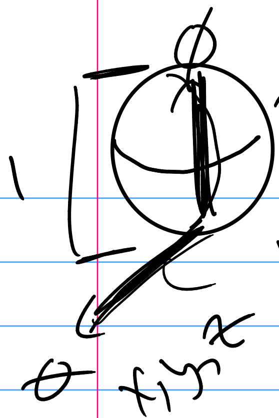
$$y = \rho \sin\phi \sin\theta$$

$$z = \rho \cos\phi$$

$$dV = dx\,dy\,dz \qquad dV = \rho^2 \sin\phi \, d\rho\, d\theta\, d\phi$$

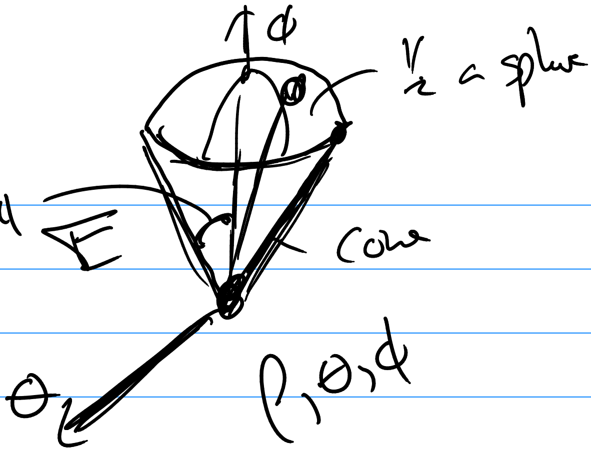






$x = \rho \sin\phi \cos\theta$
$y = \rho \sin\phi \sin\theta$
$z = \rho \cos\phi$

$\phi = \pi/4$ E



$$\iiint_E f \, dV = \int_0^{\pi/4} \int_0^{2\pi} \left[\int_{\rho=0}^{\rho=\cos\phi} f(\rho,\theta,\phi)\, \rho^2 \sin\phi \, d\rho \right] d\theta \, d\phi$$

Substitution (Calc 1)

$$\int_{x=a}^{x=b} f(x)\, dx$$

$(u = u(x)) \rightsquigarrow x = x(u)$

$dx = x'(u)\, du$

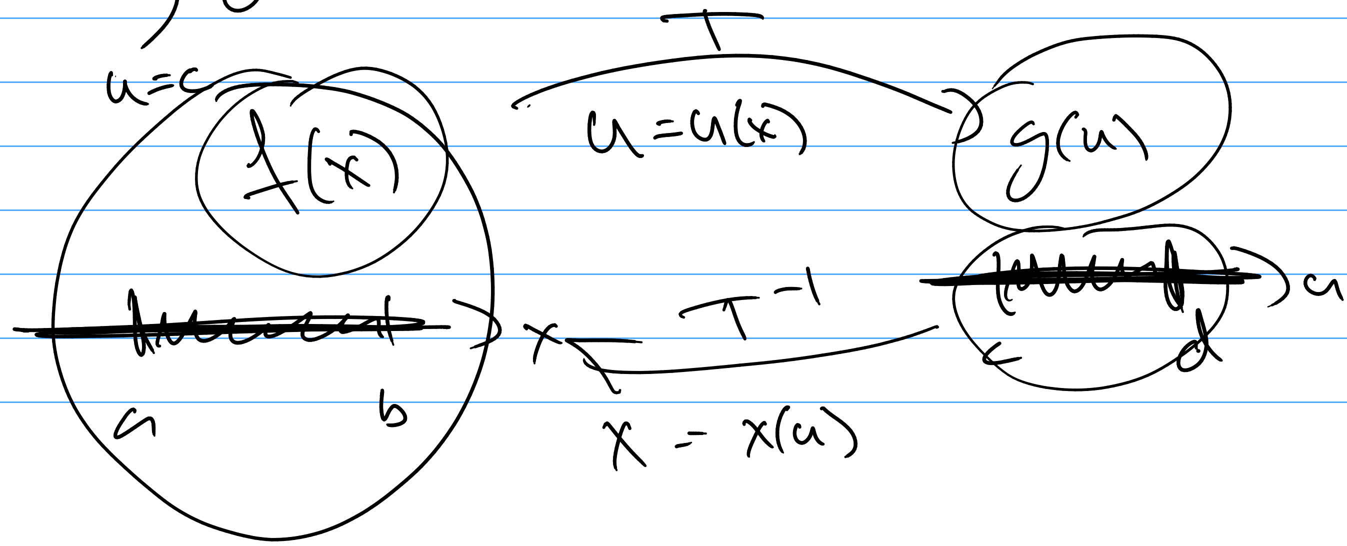
$$\int_{u=c}^{u=d} g(u)\, du$$

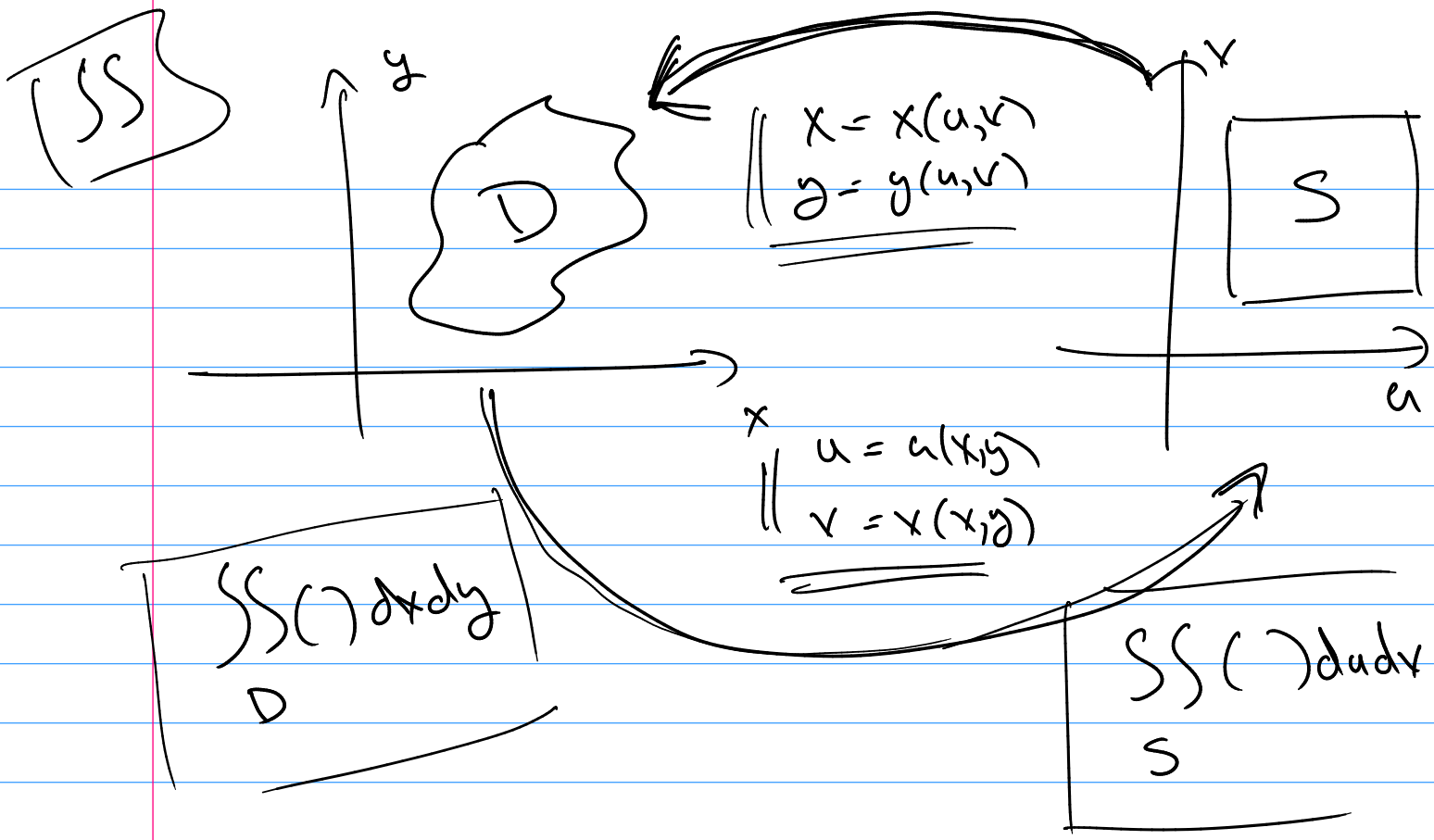
$$\int_3^4 x\sqrt{x^2+1}\, dx$$

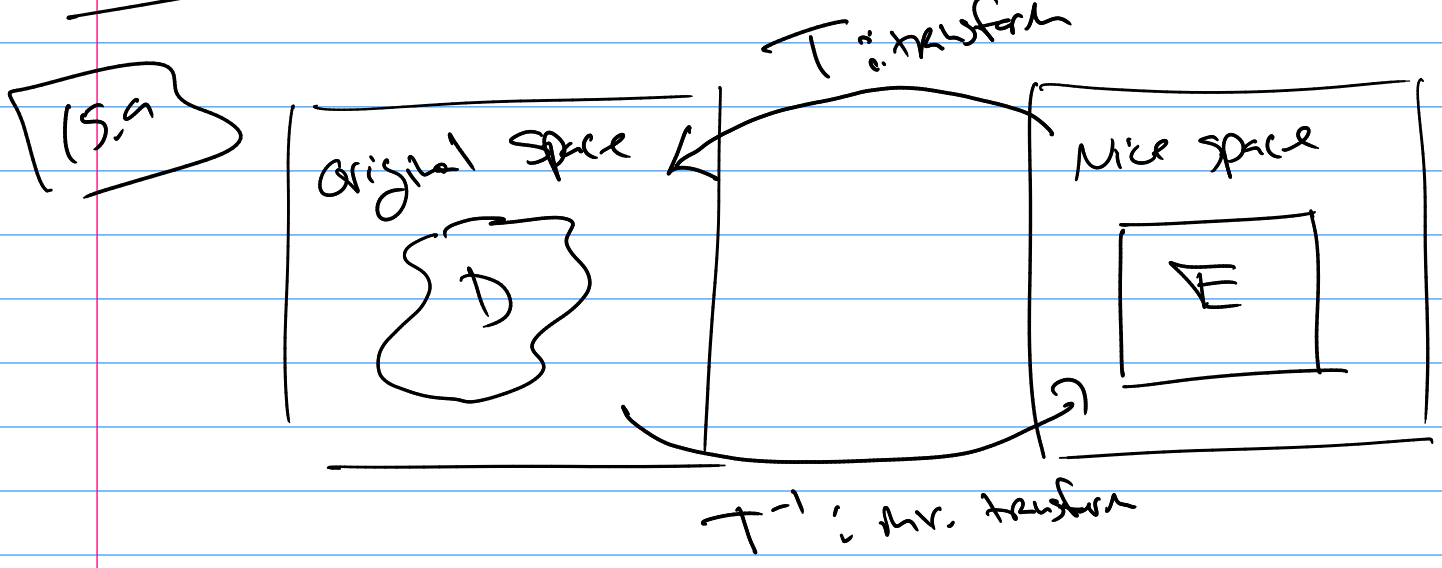
$u = x^2+1$

$du = 2x\, dx$

$$= \frac{1}{2}\int_{10}^{17} u^{1/2}\, du$$







(2D) $\iint_D (\,)\,dxdy \xrightarrow{T^{-1}} \iint_E (\,)\,dudv$

(3D) $\iiint_D (\,)\,dxdydz \xrightarrow{T^{-1}} \iiint_E (\,)\,dudvdw$

$x = \rho \sin\phi \cos\theta$