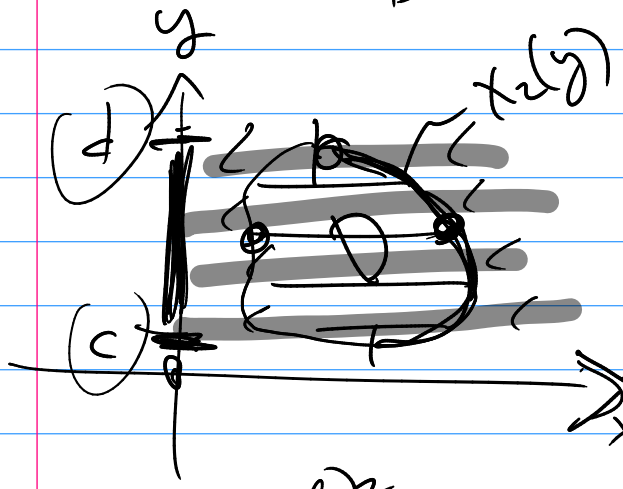
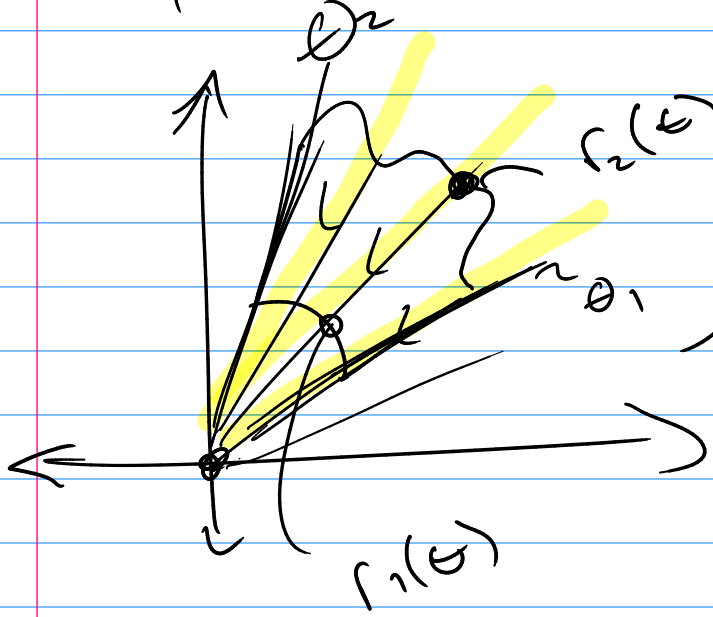
# Math 344

Bounds: $\iint_D f\,dA$ $\quad dA = dx\,dy$ $\quad dA = r\,dr\,d\theta$




$$\int_{y=c}^{y=d}\left[\int_{x=x_1(y)}^{x=x_2(y)} 1\,dx\right]dy$$




$$\int_{\theta_1}^{\theta_2}\left[\int_{r_1(\theta)}^{r_2(\theta)} f\,r\,dr\right]d\theta$$

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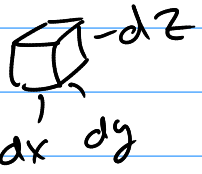
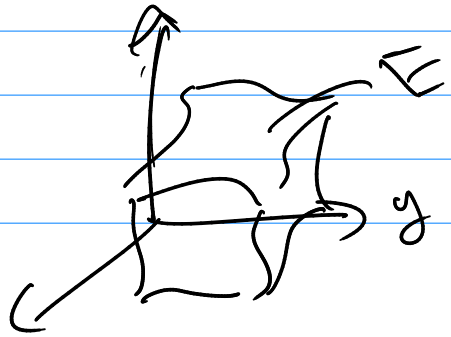
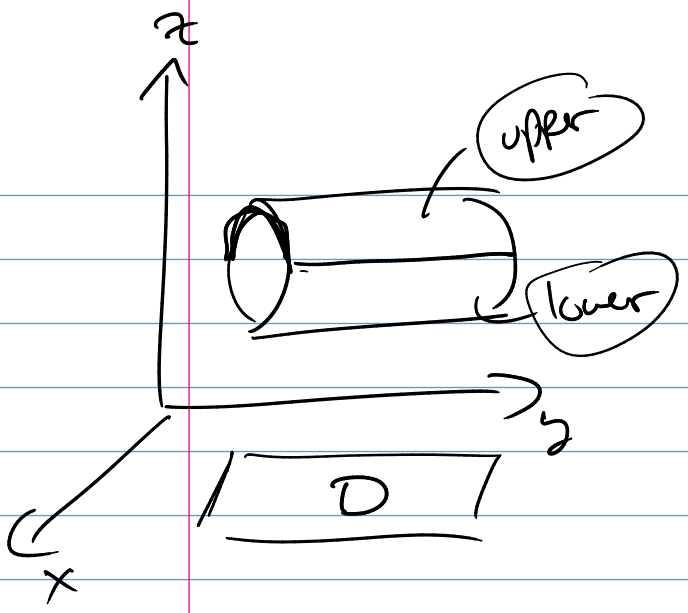
$\iiint_E A\,dV$ $\quad$ rectangular $dV$

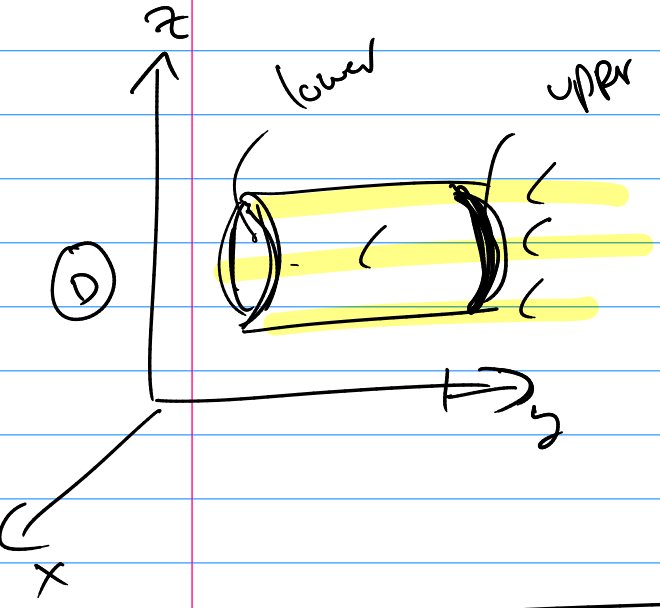



$dV = dx\,dy\,dz$






$$\iint_D \left[ \int_{\text{lower}}^{\text{upper}} 1 \, dz \right] dA \qquad z = z_2(x,y) \qquad z = z_1(x,y)$$



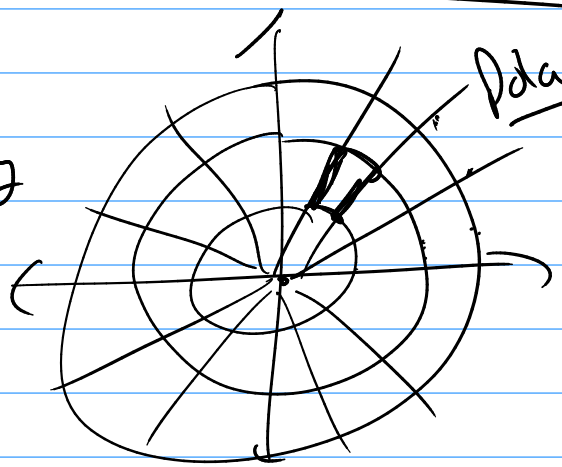

$y = \text{const.}$ for this prob

$$\iint_D \left[ \int_{\text{lower}}^{\text{upper}} 1 \, dy \right] dA$$

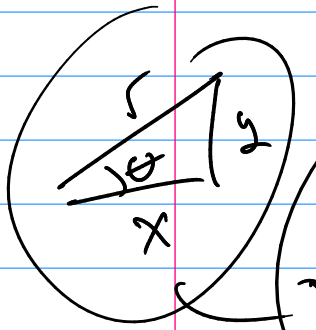
2D


rectangular

$dA = dx\,dy$



polar




$x = r\cos\theta$

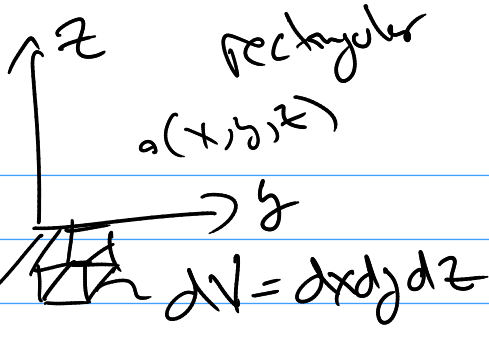
$y = r\sin\theta$

$\frac{y}{x} = \tan\theta$

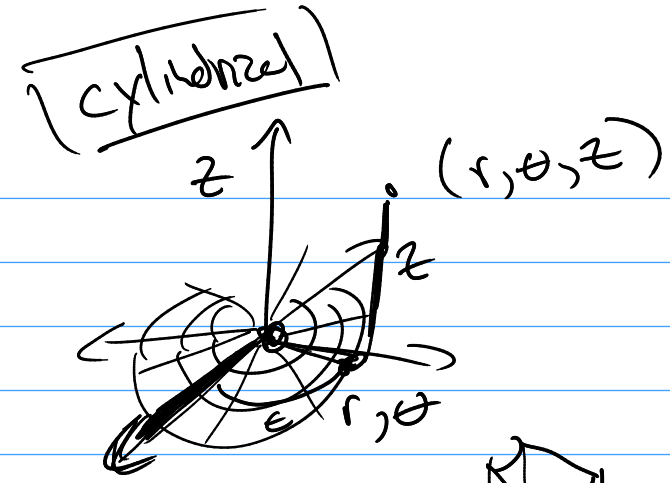
$x^2 + y^2 = r^2$

$dA = r\,dr\,d\theta$

3D



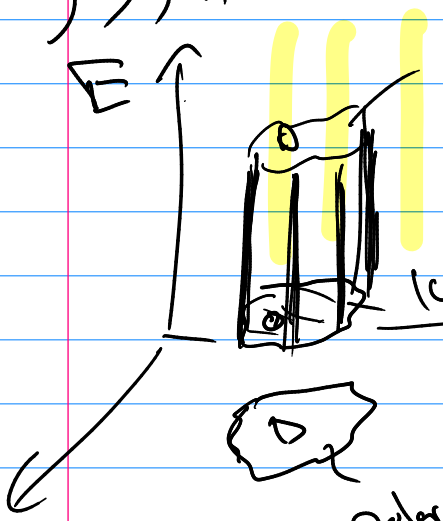

cylindrical




dV ⇒

$dV = r\,dr\,d\theta\,dz$

$\iiint_E 1\,dV$




$$\iint_D \left[ \int_{\text{lower}}^{\text{upper}} 1\,dz \right] dA$$

but in polar

$(x,y,z) \rightsquigarrow (r,\theta,z)$

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

$\frac{y}{x} = \tan\theta \qquad x^2 + y^2 = r^2$

$dV = r\,dr\,d\theta\,dz$

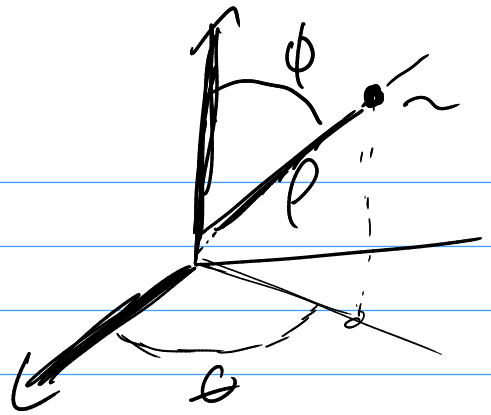
$$\int_{\theta=}^{} \int_{r=}^{} \left[ \int_{\text{lower}}^{\text{upper}} 1\,dz \right] dr\,d\theta$$

upper $z = z_2(x,y) = z_2(r\cos\theta, r\sin\theta)$

lower $z = z_1(x,y) = z_1(r\cos\theta, r\sin\theta)$

$$\int_{\theta=}^{} \int_{r=}^{} \left[ \int_{z_1(r\cos\theta, r\sin\theta)}^{z_2(r\cos\theta, r\sin\theta)} 1(r\cos\theta, r\sin\theta, z)\,dz \right] dr\,d\theta$$

Spherical




$(\rho, \theta, \phi)$

$x, y, z$

$$\iiint_E f(x,y,z)\, dV$$

$dV$

$$dV = \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$dV = (d\rho)(\rho\, d\phi)(\rho \sin\phi \, d\theta)$$