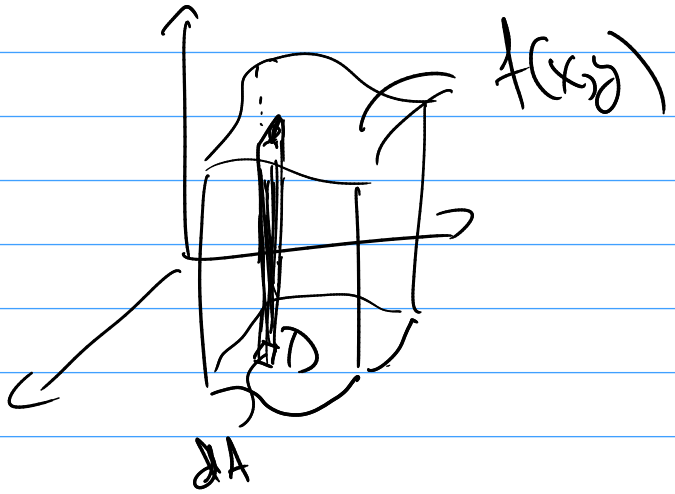
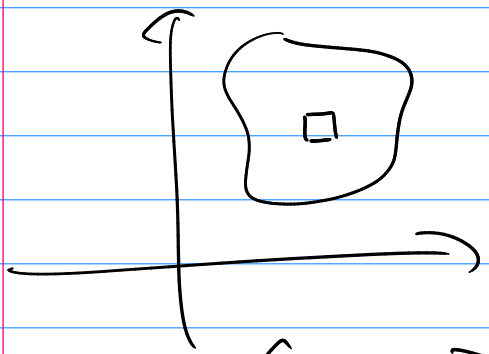
Math 344

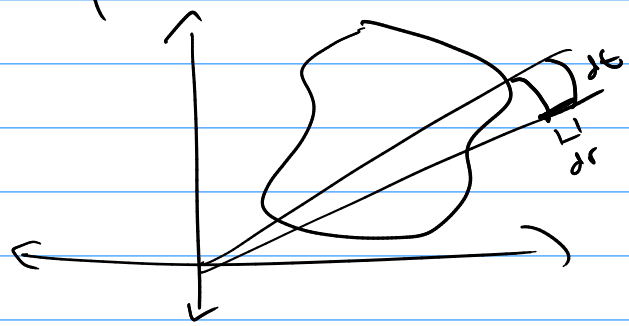
Double Integral



$$\iint_D f \, dA$$

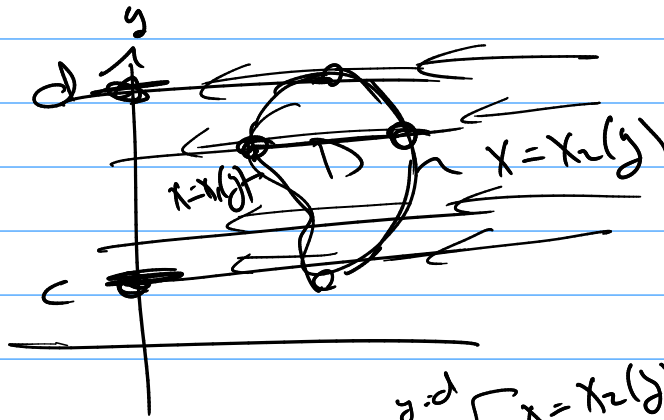


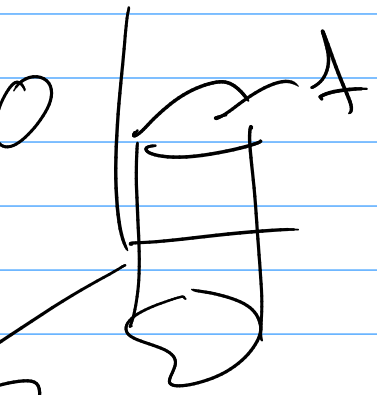
$$dA \sim \square \, dx \, dy$$



$$dA \sim \square \, dr \, d\theta$$

$$r \, dr \, d\theta$$



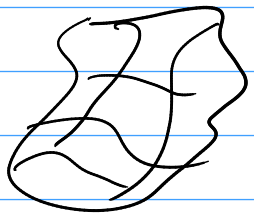


$$\int_{y=c}^{y=d} \left\{ \int_{x=x_1(y)}^{x=x_2(y)} f(x,y) \, dx \right\} dy$$

Triple Integral

$$\iiint_E f(x,y,z)\,dV$$

(ex)



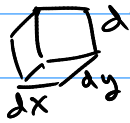
$q(x,y,z) = \frac{\text{charge}}{\text{volume}}$

$$\iiint_E q(x,y,z)\,dV$$

charge (total) = charge / ~~vol~~

$$\iiint_E f\,dV$$

rectangular $dV = dx\,dy\,dz$



Fubini's (rectangular) so $dV = dx\,dy\,dz$

(fixed cube)

E: $a \le x \le b \quad c \le y \le d \quad r \le z \le s$

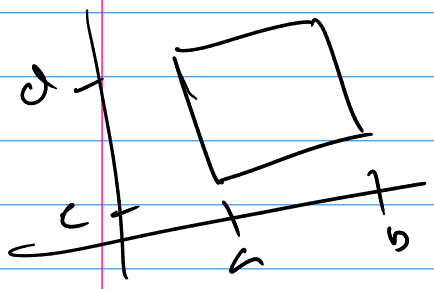
$$\iiint_E f\,dV = \int_r^s \left[\int_c^d \left[\int_a^b f\,dx\right] dy\right] dz$$

$$= \int_c^d \left[\int_r^s \left[\int_a^b f\,dx\right] dz\right] dy \quad \text{or} \quad \iint_{y,z} dA$$

$$= \int_r^s \left[\int_a^b \left[\int_c^d f \, dy \right] dx \right] dz$$

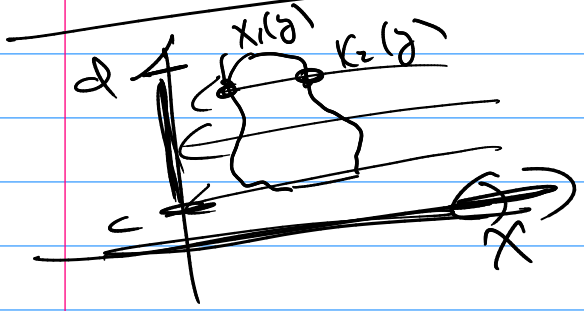
$$= \int_a^b \left[\int_r^s \left[\int_c^d f \, dy \right] dz \right] dx$$

etc



$$\iint_D f \, dA = \int_c^d \left[\int_a^b f \, dx \right] dy$$

$$= \int_a^b \left[\int_c^d f \, dy \right] dx$$



$$\int_c^d \left[\int_{x_1(y)}^{x_2(y)} f \, dx \right] dy$$

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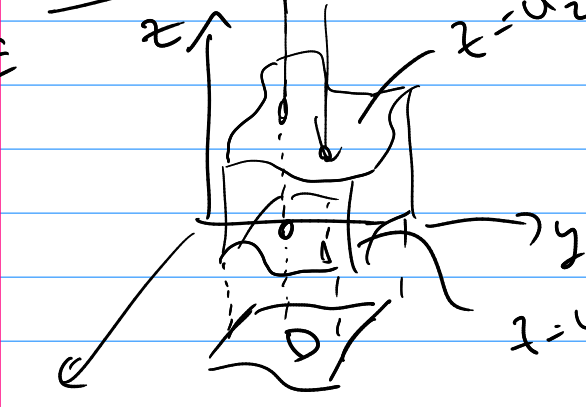
(15) $\iiint_E f(x,y,z) \, dV$



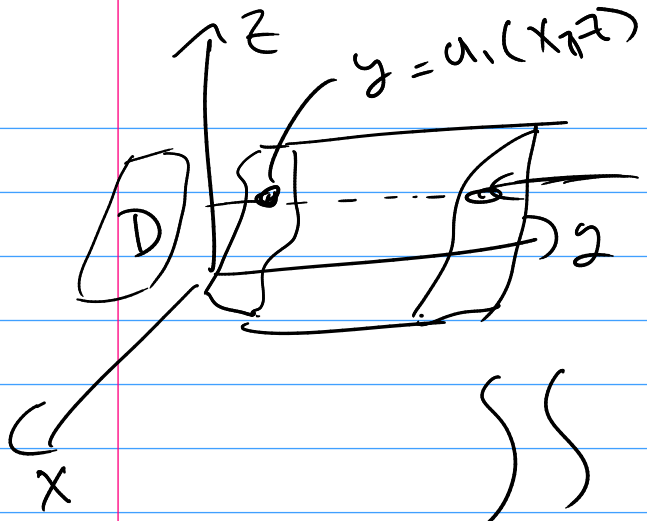
$$\iint_D \left[\int_{z=u_1(x,y)}^{z=u_2(x,y)} f \, dz \right] dA$$

$F(x,y)$

a x_1z plane

now $\iint_D F(x,y) \, dA$

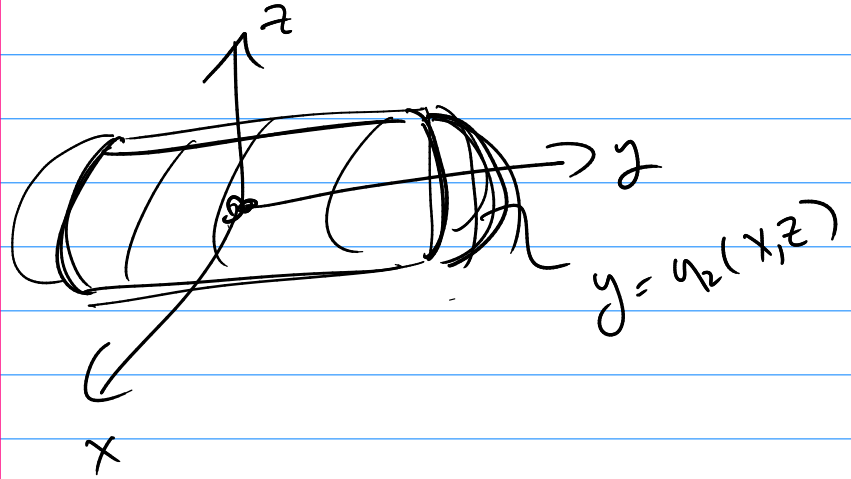
15.1
15.2



$$\iint_D \left[\int_{y=u_1(x,z)}^{y=u_2(x,z)} f(x,y,z)\,dy \right] dA$$

dA : xz plane

$F(x,z)$



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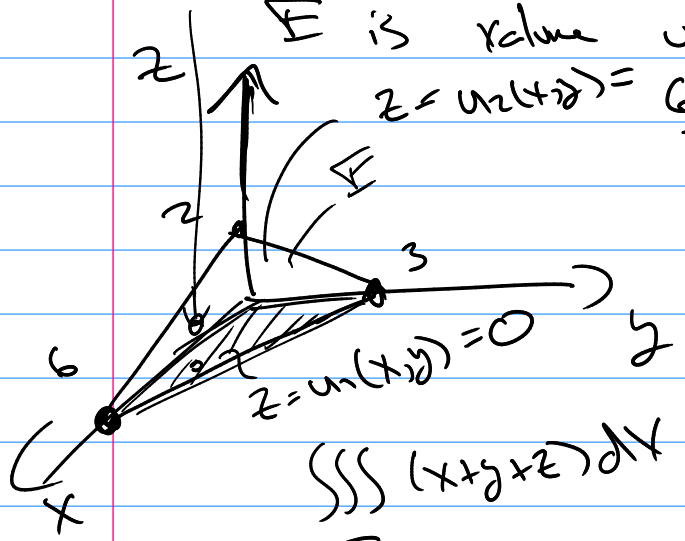
plane $x + 2y + 3z = 6$

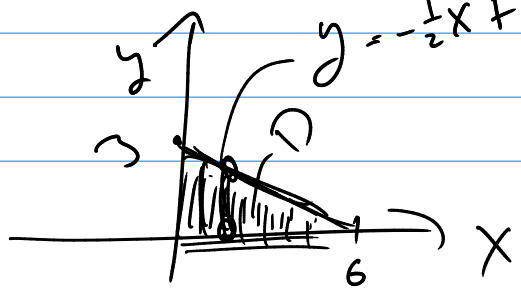
E is region under the plane in 1st octant

$z = u_2(x,y) = \dfrac{6 - x - 2y}{3}$



$$\iiint_E (x+y+z)\,dV$$

$$\iint_D \left[\int_{z=0}^{z=2-\frac{1}{3}x-\frac{2}{3}y} (x+y+z)\,dz \right] dA$$



$$\iiint_E (x+y+z)\,dV = \int_{x=0}^{x=6}\left[\int_{y=0}^{y=-\frac{1}{2}x+3}\left[\int_{z=0}^{z=2-\frac{1}{3}x-\frac{2}{3}y}(x+y+z)\,dz\right]dy\right]dx$$

$$= \int_{x=0}^{x=6}\left[\int_{y=0}^{y=-\frac{1}{2}x+3}\left[(x+y)z+\frac{1}{2}z^2\Big|_{z=0}^{z=2-\frac{1}{3}x-\frac{2}{3}y}\right]dy\right]dx$$

(*)

(*) $(x+y)\left(2-\frac{1}{3}x-\frac{2}{3}y\right)+\frac{1}{2}\left(2-\frac{1}{3}x-\frac{2}{3}y\right)^2$

$=$

$=$;

$= F(x,y)$