Math 344

Exam 2

#7

~~$x^2+y^2=16$~~

~~$x^2+x^2=16$~~

x^2+y^2

$\nabla f \cdot \hat{u}$

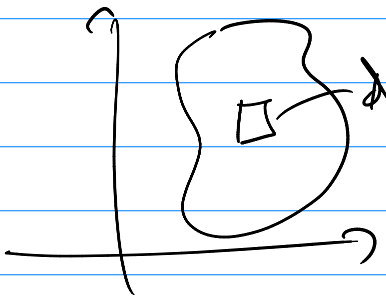
$D_u(f)$ ~~at (9,9)~~

$$D_u(f) = \frac{1}{10\sqrt{xy}}(3y-4x)$$

$$\frac{1}{10\sqrt{81}}(3\cdot 9-4\cdot 9) = -\frac{1}{10}$$

$$\iint_D f\,dA$$

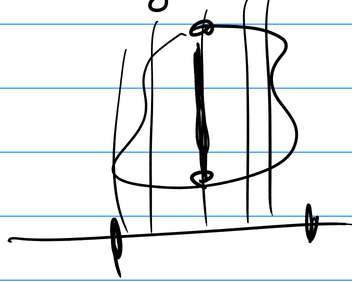
15.1 to 15.3

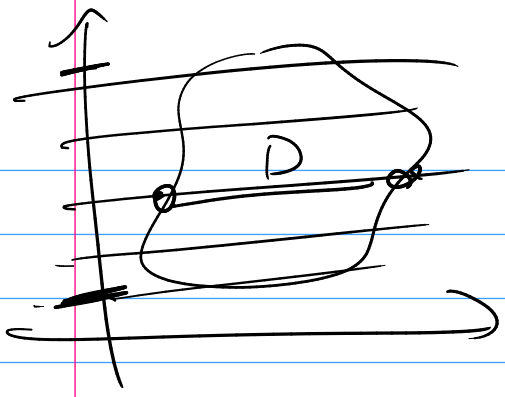


$dA = dx\,dy$

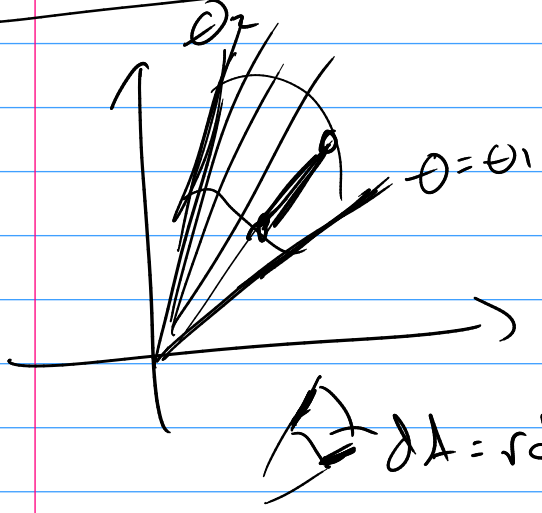
$= dy\,dx$

$$\int_{x=}^{x=}\left[\int_{y=}^{y=} f\,dy\right]dx$$





$$\int_{y=}^{y=} \left[\int_{x=}^{x=} f \, dx \right] dy$$



$$\int_{\theta=\theta_1}^{\theta=\theta_2} \left[\int_{r=}^{r=} f(r\cos\theta, r\sin\theta)\, r\, dr \right] d\theta$$

$x \quad y$

15.1 Applications

Physics → (change) is important

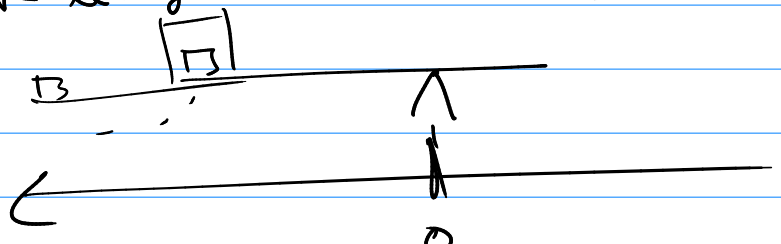
Momentum: change linear motion ≡ force

(M·V)

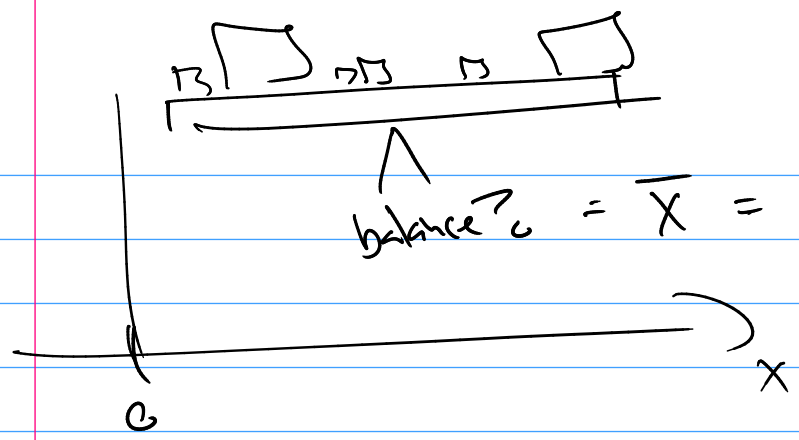
need more force for more mass
also for more velocity

levers

more leverage for more mass also for more length



Moment = (mass)(length)



$$\bar{x} = \frac{\text{Sum all moments}}{\text{Sum all mass}}$$

Mass across D is variable

use $\rho(x,y)$ for that fact

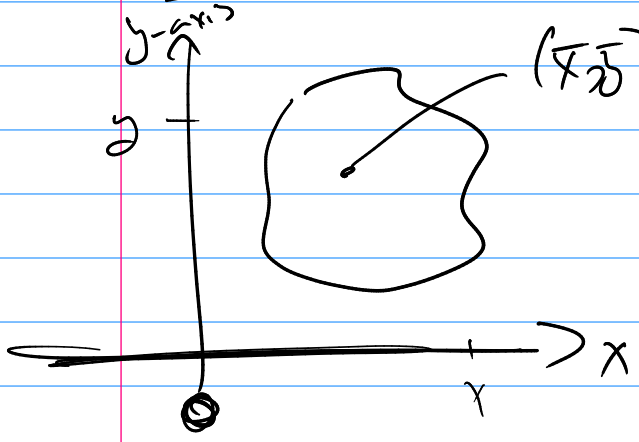
$$\text{density} = \rho(x,y) \quad \frac{\text{mass}}{\text{area}}$$

total mass:

$$M = \iint_D \rho(x,y)\,dA$$

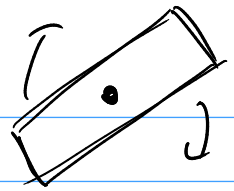
center of Mass?

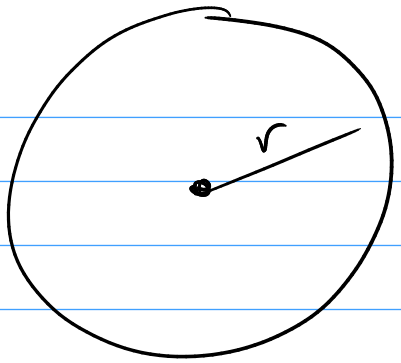
$$M_{y\text{-axis}} = \iint_D x\,\rho(x,y)\,dA$$



$$\bar{x} = \frac{\iint_D x\,\rho(x,y)\,dA}{\iint_D \rho(x,y)\,dA} = \frac{M_{y\text{-axis}}}{M}$$

$$\bar{y} = \frac{\iint_D y\,\rho(x,y)\,dA}{\iint_D \rho(x,y)\,dA} = \frac{M_{x\text{-axis}}}{M}$$

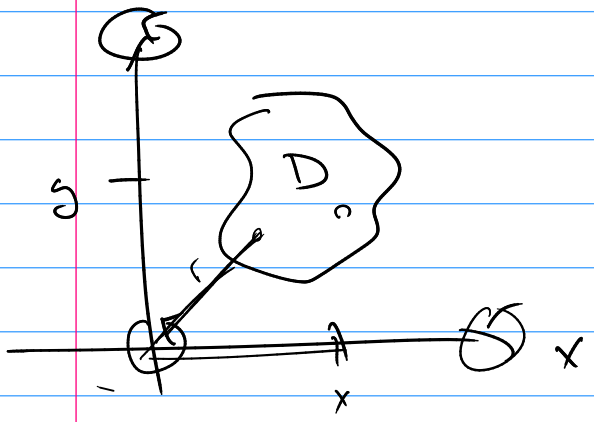




change rate of rotation

more torque means more mass also more r^2

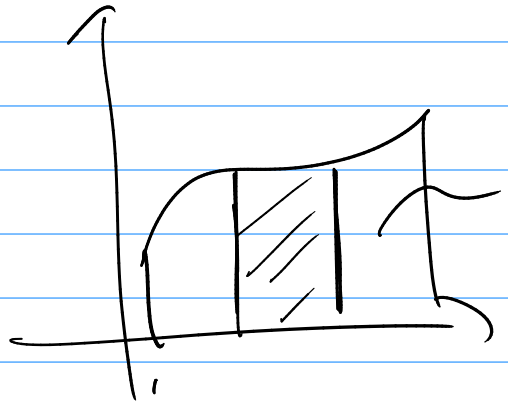
Moment of Inertia $\sim mr^2$



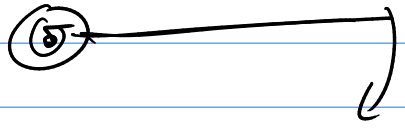
$$I_{y\text{-axis}} = \iint_D x^2 \rho(x,y)\,dA$$

$$I_{x\text{-axis}} = \iint_D y^2 \rho(x,y)\,dA$$

$$I_{origin} = \iint (x^2+y^2)\rho(x,y)\,dA = I_{y\text{-axis}} + I_{x\text{-axis}}$$



area = 1 = 100%



axle