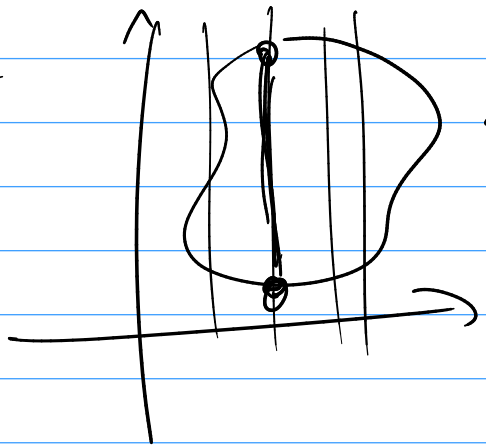
Math 314

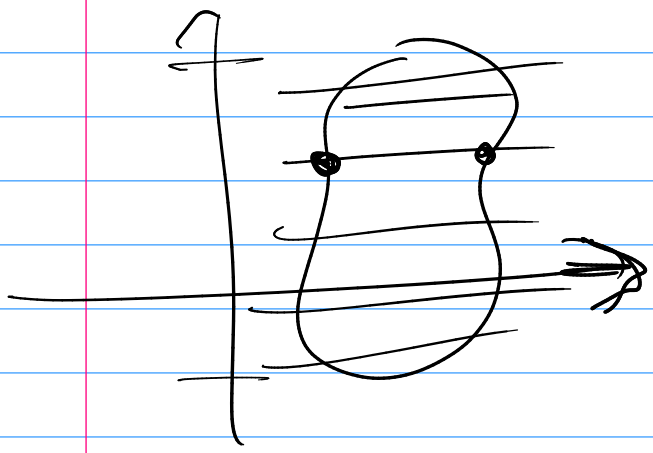
Q's

15.1 / 15.2 HW due 2 days after Wed.

15.1 / 15.2

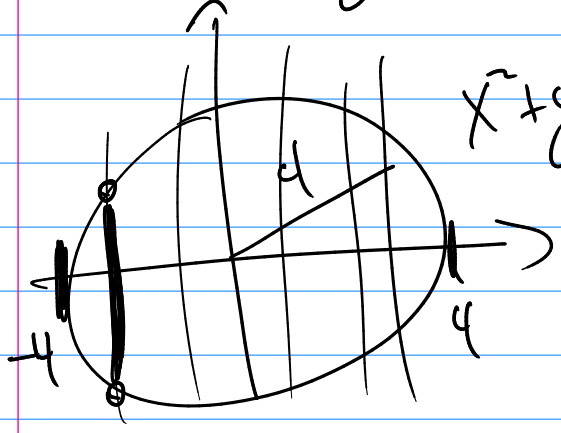


$$\int_{x=}^{x=} \left[\int_{y=}^{y=} 1 \, dy \right] dx$$



$$\int_{y=}^{y=} \left[\int_{x=}^{x=} 1 \, dx \right] dy$$

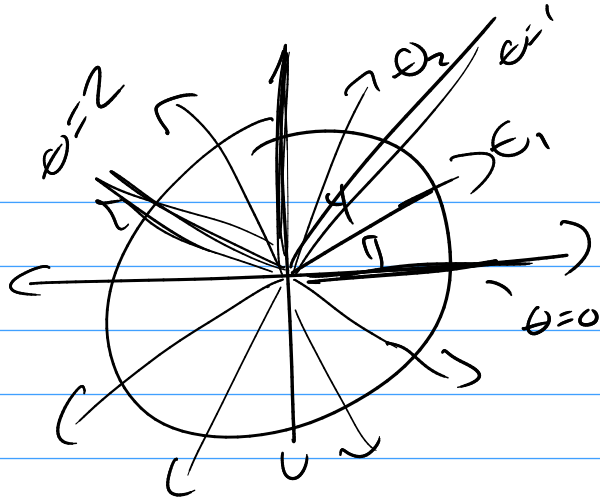
For some regions polar representation "better".



$x^2 + y^2 = 16$

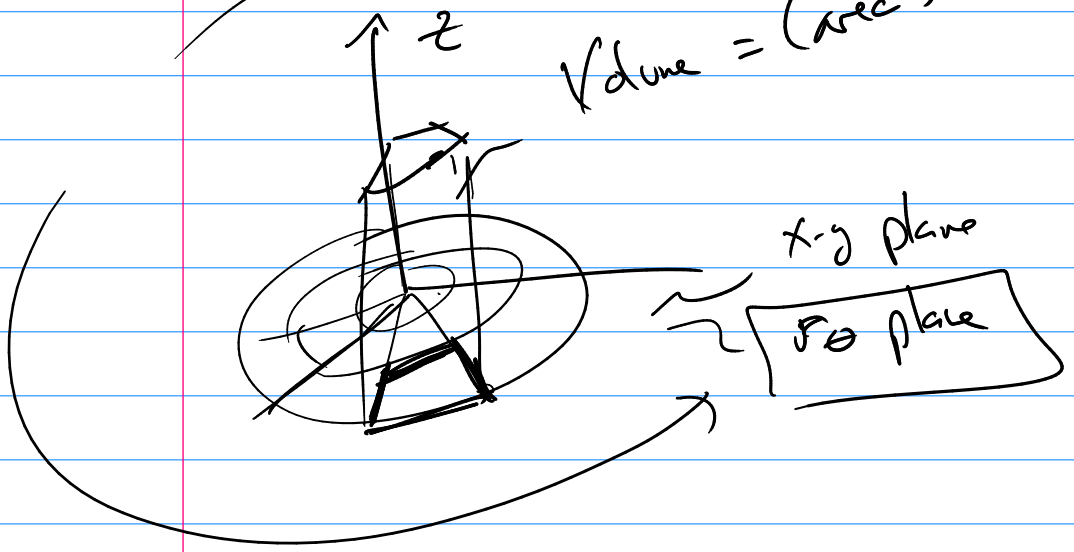
$$\int_{x=-4}^{x=4} \left[\int_{y=-\sqrt{16-x^2}}^{y=\sqrt{16-x^2}} 1(\,) \, dy \right] dx$$

in polar



and θ is anything from 0 to 2π

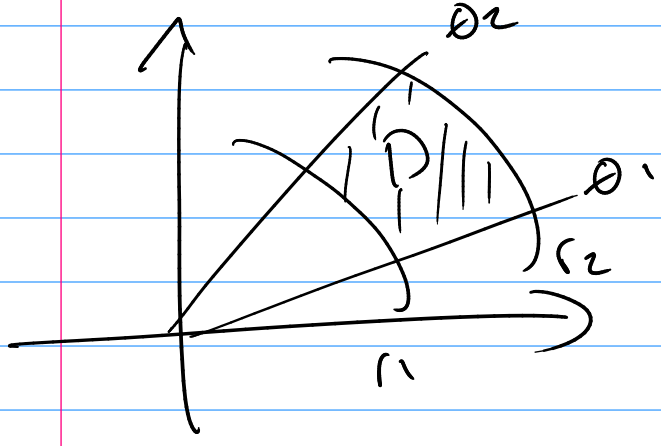
3D (cylindrical)



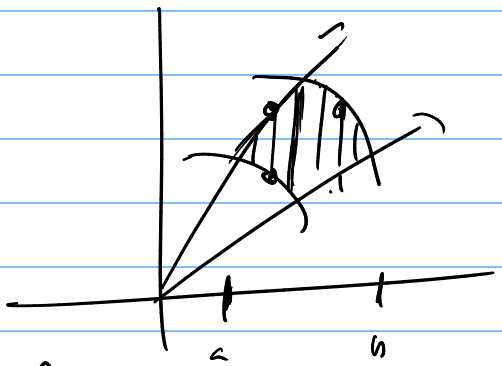
Volume = (area base)(height)

x-y plane

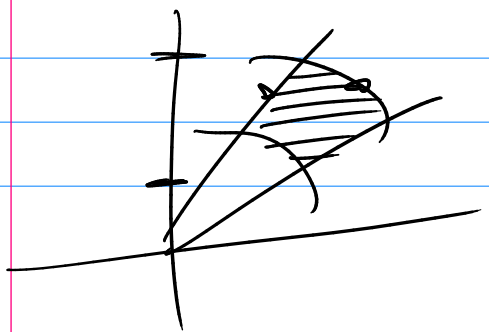
$r\theta$ plane



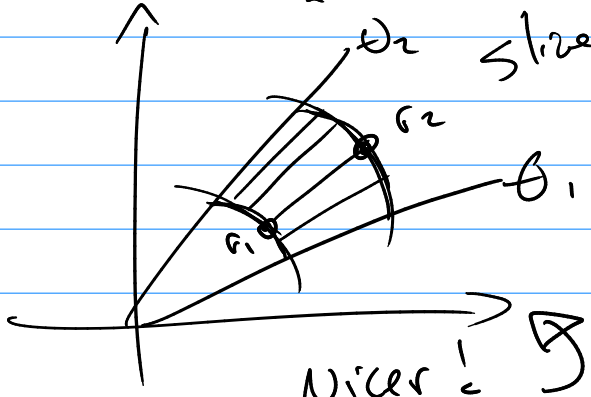
slice by x



slice by z



slice by θ

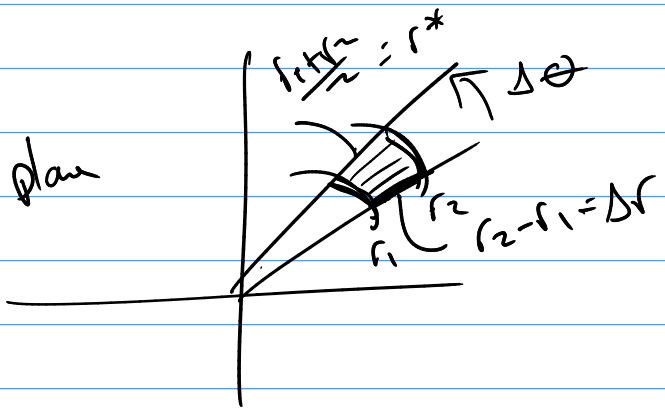


Nicer!

$$Vol = \iint_D \boxed{h\, dA} \quad \text{(height)(area)}$$

h = dA from ΔA

$r\theta$ plane



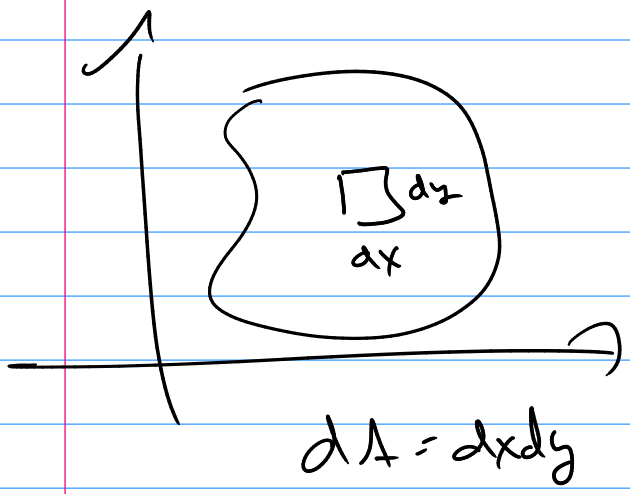
$$\Delta A = \frac{1}{2} r_2^2 \Delta\theta - \frac{1}{2} r_1^2 \Delta\theta$$

$$\Delta A = \frac{1}{2}\left(r_2^2 - r_1^2\right)\Delta\theta$$

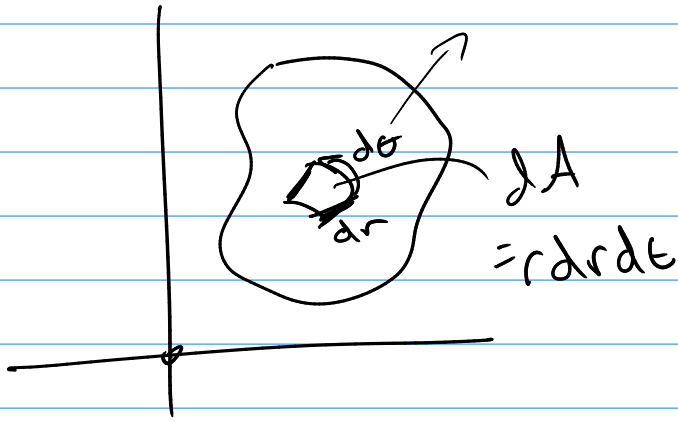
$$\Delta A = \frac{1}{2}\left(r_2 + r_1\right)\underbrace{\left(r_2 - r_1\right)}_{\Delta r}\Delta\theta$$

$$\Delta A = r^* \Delta r \Delta\theta$$

$$\Rightarrow dA = r\, dr\, d\theta$$



$dA = dx\,dy$



So $$Vol = \iint_D f \, dA$$

If D is in <u>polar coord</u>

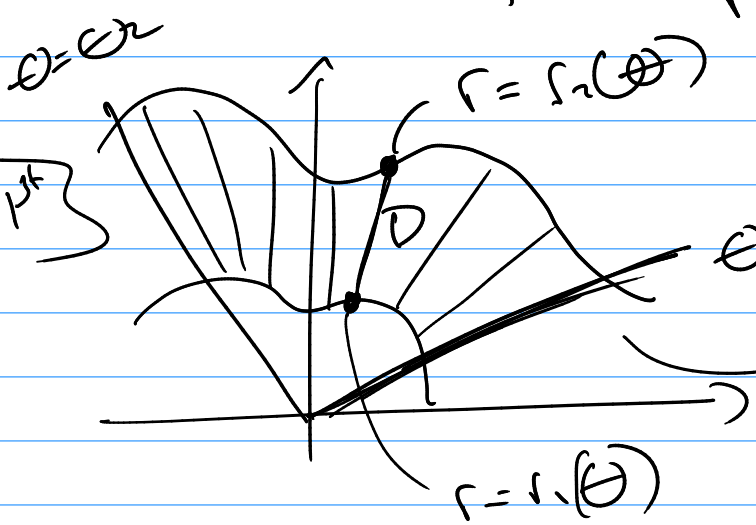
$$Vol = \iint_D f \underbrace{(r \, dr \, d\theta)}_{dA}$$

two things new ① D in polar form

② f in polar $f(r,\theta)$

How?

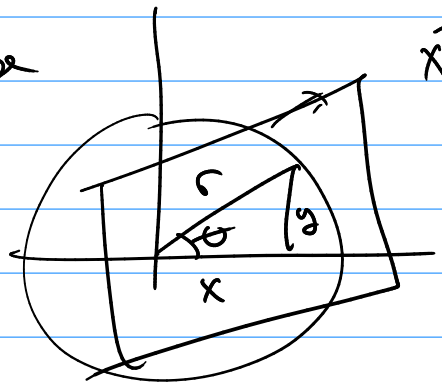
1st



$$V = \int_{\theta_1}^{\theta_2} \left[\int_{r_1(\theta)}^{r_2(\theta)} f(r,\theta) \, r \, dr \, d\theta \right]$$

2nd $f(x,y) \Rightarrow f(r,\theta)$

use



$x^2 + y^2 = r^2$

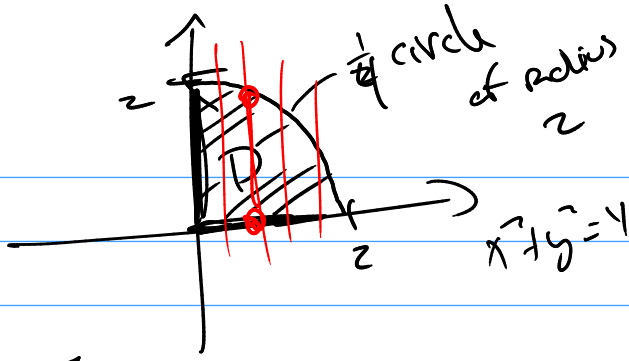
$x = r\cos\theta$

$y = r\sin\theta$

$\frac{y}{x} = \tan\theta$

etc

(ex) $\iint_D (x^2+y^2)\,dA$

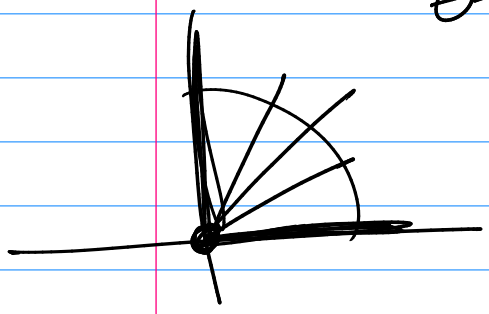


15.1 | 15.2 (rectangular)

$$Vol = \int_{x=0}^{x=2}\left[\int_{y=0}^{y=\sqrt{4-x^2}} (x^2+y^2)\,dy\right]dx$$

15.3

$$Vol = \int_{\theta=0}^{\theta=\pi/2}\left(\int_{r=0}^{r=2} \underbrace{(x^2+y^2)}_{r^2} r\,dr\right)d\theta$$



$$= \int_0^{\pi/2}\left(\int_0^2 r^3\,dr\right)d\theta$$