Math 314

Exam 12 probs @ 10pts

110pts = 100%

open | all day wednesday

2 hrs to finish

Ch 14 "Change" for $u = f(x_1, x_2, \ldots, x_n)$

14.1 / 14.2 - Intro (graph)
- limits
(no probs)

14.3 Partial Derivatives (2 probs)

① 1st partials

x^2yz^{-1}

ex $u = 3xyz + \frac{x^2y}{z} - \cos(yz)$

$u_x = 3yz + 2xyz^{-1}$ → Don't simplify

$u_y = ?$

$u_z = ?$

(2) Higher partials $u = f(x,y)$

(cb) 2nd partials check $f_{xy} = f_{yx}$

3rd partial $f_{xyx} = ?$

(ex) $f(x,y) = \sqrt{xy} + \sec(xy)$ $\quad \sqrt{xy} = x^{1/2} y^{1/2}$

$$f_x = \frac{1}{2} y^{1/2} x^{-1/2} + \sec(xy)\tan(xy)\, y$$

$$f_{xy} = \frac{1}{4} y^{-1/2} x^{-1/2} + (x\sec(xy)\tan(xy))(\tan(xy))\, y + \sec(xy)\left[x\sec^2(xy)\, y + \tan(xy)\cdot 1\right]$$

14.4 | 2 probs

(1) use $f(x,y) \approx f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

near (a,b)

(ex) $f(x,y) = \sin(x) + \cos(y)$

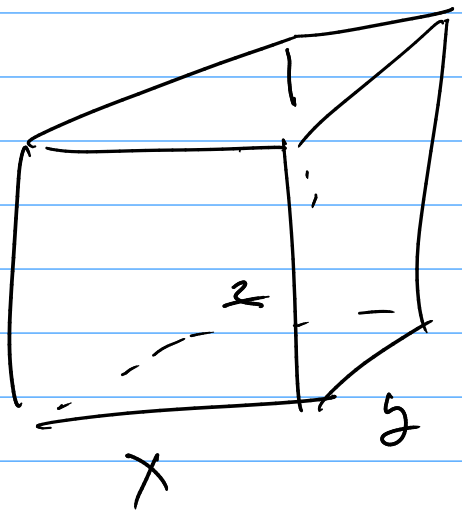
find $f(3, 1.5)$ near $f(\pi, \pi/2)$

a, b

(2) $u = f(x,y,z)$ $dx = \Delta x$

$$\Delta u \approx du = f_x dx + f_y dy + f_z dz$$

$x = 2,\ y = 3,\ z = 1$ ← $\Delta x, \Delta y, \Delta z$ are all ± 0.1



$S = xy + 2xz + 2yz$

$$\Delta S \approx dS = S_x dx + S_y dy + S_z dz$$

$= ?$

0

11.5 (2 probs) chain rule

① (ex) $z = f(x,y)$ $x(t), y(t)$

$$\frac{dz}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt}$$

(ex) $u = xyz^2$ $x = 3t^2$, $y = \sqrt{t}$, $z = t^3 - 1$

by chain rule

$$\frac{du}{dt} = u_x \frac{dx}{dt} + u_y \frac{dy}{dt} + u_z \frac{dz}{dt}$$

$$\frac{du}{dt} = (yz^2)(6t) + (xz^2)\left(\frac{1}{2\sqrt{t}}\right) + (2xyz)(3t^2)$$

Stop here

② (ex) $z = f(x,y)$ $x(t,s)$, $y(t,s)$

use chain rule. $\frac{\partial z}{\partial t} = ?$ $\frac{\partial z}{\partial s} = ?$

14.6 Directional Derivatives know $D_{\vec{u}}(f)$

(2 probs) $= \nabla f \cdot \vec{u}$

① find $D_{\vec{u}}(f) = ?$

② find Max and direction & max for

$D_{\vec{u}}(f)$

14.7 extrema (2 probs)

① Find rel. extrema for $z = f(x,y)$

ex $z = x^2 y^2$

(a) find critical locations

$$\left.\begin{array}{ll} f_x = 0 & 2xy^2 = 0 \\ f_y = 0 & 2yx^2 = 0 \end{array}\right\} \text{sol. (s) } \begin{array}{l} x=0 \\ y=0 \end{array}$$

(b) 2nd partials test

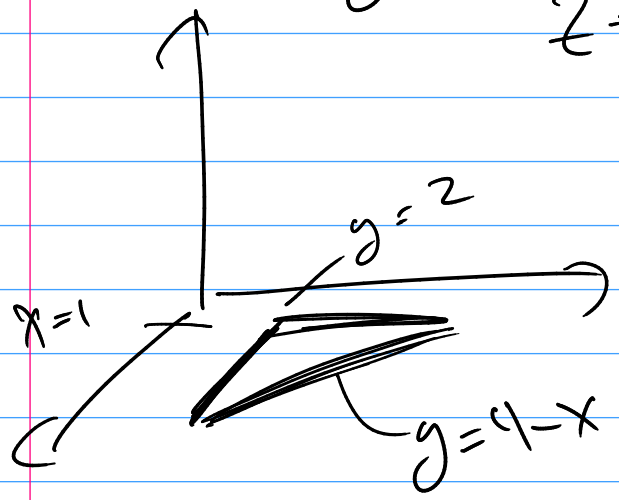
② critical locations $D = f_{xx} f_{yy} - (f_{xy})^2$

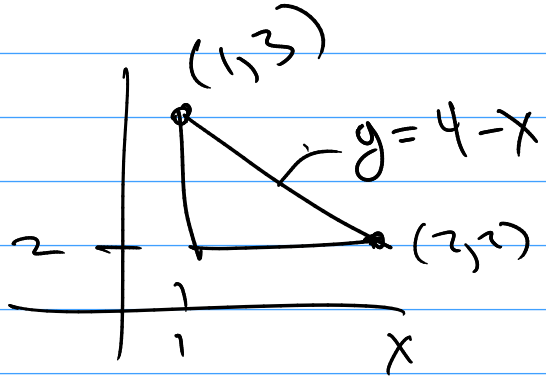
$D > 0 \rightarrow ?$

$D < 0 \rightarrow ?$

$D = 0 \rightarrow ?$

② Find abs. extrema over triangular region $z = f(x,y)$





14.8 (2 probs) $\begin{cases} \nabla f = \lambda \nabla g \\ g = k \end{cases}$

① $f(x,y)$, $g(x,y) = x$ Solve

② $f(x,y,z)$, $g(x,y,z) = k$ setup

$f(x,y,z) = \sec(xyz)$

constraint $x^2 + y^2 + z^2 = 4$

g

Solve $\begin{cases} yz \sec(xyz)\tan(xyz) = 2\lambda x \\ xz \sec(xyz)\tan(xyz) = 2\lambda y \\ xy \sec(xyz)\tan(xyz) = 2\lambda z \\ x^2 + y^2 + z^2 = 4 \end{cases}$