# Math 314

$$x_{n+1} = f(x_1, x_2, \ldots, x_n)$$

a) point $(x_1, x_2, \ldots, x_n, x_{n+1})$

$x_1, x_2, \ldots, x_n$: independent; $x_{n+1}$: dependent

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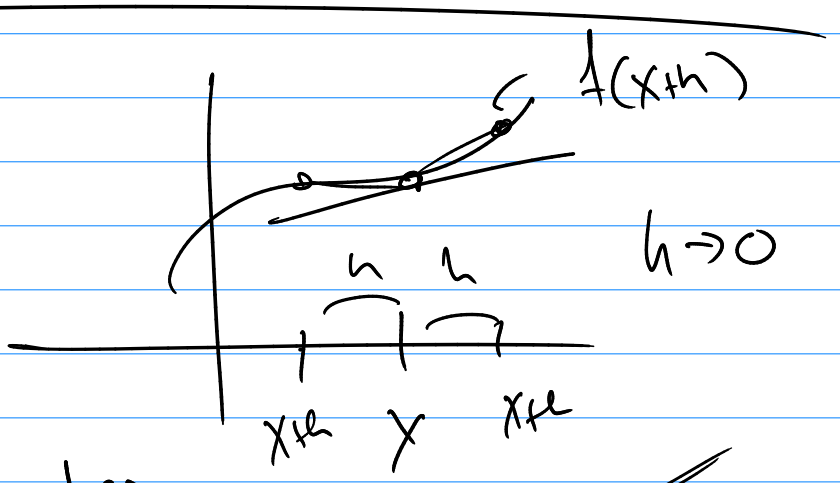
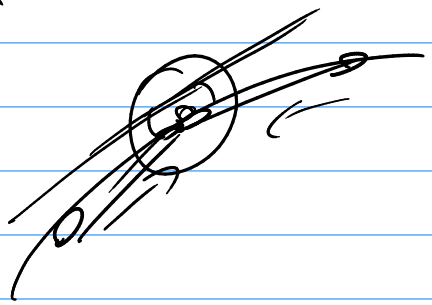
**change?**

**Calc**




$h \to 0$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



(ex) $f(x) = 3x^2 + 2x - 1$

$f(x) = ax^2 + bx + c$

$$f'(x) = \lim_{h \to 0} \frac{(a(x+h)^2 + b(x+h) + c) - (ax^2 + bx + c)}{h}$$

$$= 2ax + b$$

explicit deriv.

$$\frac{d}{dt}\left[a + b\sin(ct+1) - t\sqrt{t^2-e}\right]$$

$$= 0 + cb\cos(ct+1) - \left[(1)\sqrt{t^2-e} + t\,\frac{2t}{2\sqrt{t^2-e}}\right]$$

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Implicit deriv

$$\frac{d}{dx}\left[x + xy + y^2\right] = 1 + \left[y + xy'\right] + 2yy'$$

$\frac{d}{dx}$ but $y = f(x)$

$$x + \left[x\,\frac{\tan^3(3x^2-1)}{\sqrt{x^2+1}}\right] + \left(\frac{\tan^3(3x^2-1)}{\sqrt{x^2+1}}\right)^2$$

($x$ = $y$, $y$ = $y$)

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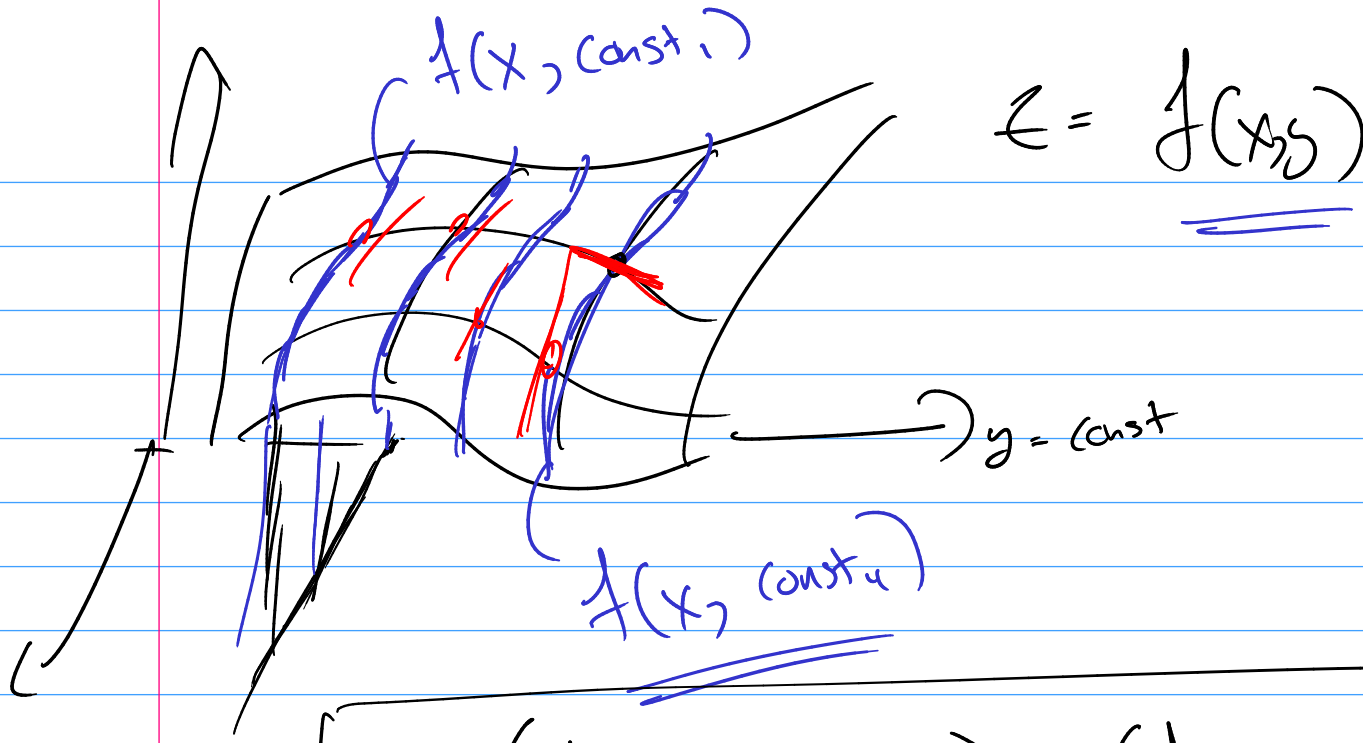
What does the above mean for

$$x_{n+1} = f(x_1, x_2, \ldots, x_n)$$

(ex) $\quad \epsilon = 3x^2 - \sin(xy) + y^3$

$$x_4 = x_1 x_2 x_3 - \sqrt{x_3^2 + x_2^2}$$




$$\frac{\partial}{\partial x} f = \lim_{h \to 0} \frac{\left(f(x+h, y)\right) - \left(f(x, y)\right)}{h}$$

$f(x,y)$ → $\frac{\partial}{\partial x} f$ ( partial derivative of $f$ with respect to $x$. [hold $x$ as variable all other ind. variables are thought of as const])

$$\frac{\partial}{\partial x}[f] = f_x = z_x = D_x[f]$$

Calc Notation:

$$\frac{d}{dx}[f] = f' = D[f]$$

$$\int f\,dx = A[f]$$

ex

## 1st partials

$f(x,y,z) = xy + z^2$

$f_x = D_x[f]$ $\quad$ $f_x = y = f_x(x,y,z)$

$f_y = D_y[f]$ $\quad$ $f_y = x = f_y(x,y,z)$

$f_z = D_z[f]$ $\quad$ $f_z = 2z = f_z(x,y,z)$

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$f(x,y)$

$\frac{\partial}{\partial x}$ $\quad$ $\frac{\partial}{\partial z}$

1st partials: $f_x(x,y)$ $\quad$ $f_y(x,y)$

$\frac{\partial}{\partial x}$ $\quad$ $\frac{\partial}{\partial y}$ $\quad$ $\frac{\partial}{\partial y}$

2nd partials: $f_{xx}(x,y)$ $\quad$ $f_{xy}(x,y)$ $\quad$ $f_{yx}(x,y)$ $\quad$ $f_{yy}(x,y)$

Notation: $f_{xy} = \frac{\partial}{\partial y}\left[\frac{\partial}{\partial x}[f]\right]$

ex $f_{xxzy} = \frac{\partial}{\partial y}\left[\frac{\partial}{\partial z}\left[\frac{\partial}{\partial x}\left[\frac{\partial}{\partial x}[f]\right]\right]\right]$

$f(x,y,z) = xyz$

1st: $f_x = yz$ $\quad f_y = xz$ $\quad f_z = xy$

2nd: $f_{xx} = 0$ $\quad f_{xy} = z$ $\quad f_{xz} = y$

$3x^2 + 1 = 0$

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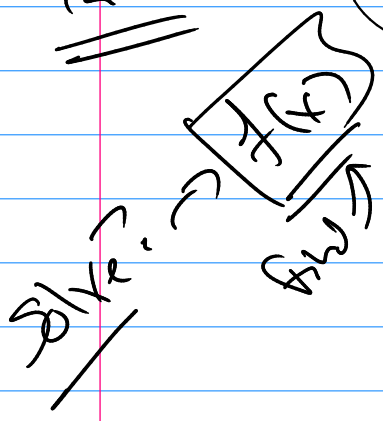
Calc I

Differential Equations




$f'' + f = 0$

$f - f' = 0$

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Partial Differential Equations

(ex) Laplace Eqn: $u(x,y)$

$u_{xx} + u_{yy} = 0$