Stat873 HW10

1. Consider $Y \sim N(X\beta, \sigma^2 I_n)$ where $X \in R^{n\times p}$ and $1_n \in \mathcal{R}(X)$.

   (1) Show that $SSM = Y'(XX^+ - 1_n 1_n^+)Y$ is an SS.

   $1_n \in \mathcal{R}(X) \Longrightarrow 1_n = Xh$ for some h. So $XX^+ 1_n 1_n^+ = 1_n 1_n^+ = 1_n 1_n^+ XX^+$.
   It follows that $XX^+ - 11^+$ is symmetric and idempotent.
   Thus $XX^+ - 11^+ = (XX^+ - 11^+)(XX^+ - 11^+)^+$.
   Hence $SSM = Y'(XX^+ - 11^+)Y$ is an SS.

   (2) Show that SSM is part of SSTO$= Y'(I - 11^+)Y$.

   By $XX^+ 11^+ = 11^+ = 11^+ XX^+$,
   $(I - 11^+)(XX^+ - 11^+) = XX^+ - 11^+ - 11^+ + 11^+ = XX^+ - 11^+$.
   Thus $SSM = Y'(XX^+ - 11^+)Y$ is part of $SSTO = Y'(I - 11^+)Y$.

   (3) Find an SS such that $SSTO = SSM + SS$.

   Let SS be $Y'(I - XX^+)Y$.
   Because $I - XX^+ = (I - XX^+)(I - XX^+)^+$, $Y'(I - XX^+)Y$ is an SS.
   Clearly $SSTO = Y'(I - 11^+)Y = Y'(XX^+ - 11^+)Y + Y'(I - XX^+)Y = SSM + SS$.

2. For model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon$ consider a test on $H_0 : \begin{pmatrix} -0.5 & 10 & 0 & -20 \\ 0 & 0 & 1 & 0 \end{pmatrix} \beta = 0$.
   Write your report with y, x1, x2, x3 in file "ex.txt".

```
data a;
    infile 'D:\ex.txt";
    input y x1 x2 x3;
proc reg;
    model y=x1 x2 x3/noprint;
    test -0.5*intercept+10*x1-20*x3=0, x2=0;
    run;
```

$H_0 : \begin{pmatrix} -0.5 & 10 & 0 & -20 \\ 0 & 0 & 1 & 0 \end{pmatrix} \beta = 0$ vs $H_a : \begin{pmatrix} -0.5 & 10 & 0 & -20 \\ 0 & 0 & 1 & 0 \end{pmatrix} \beta \neq 0$
Test statistic: $F = \frac{MSD}{MSE}$
p-value: $P(F(2, n-4) > F_{ob})$

$F_{ob} = \frac{0.25978}{12.54508} = 0.02$
p-value: $P(F(2, 32) > 0.02) = 0.9795$.

Fail to reject H_0.