Stat873 **HW07**

1. Consider two-sided t-tests

   (1) Complete the α-level test scheme.

   $H_0 : l'\beta = b$ vs $H_a : l'\beta \neq b$
   Test Statistic: $t = \frac{l'\widehat{\beta} - b}{S_{l'\widehat{\beta}}}$
   Reject H_0 if $t < -t_{\alpha/2}(n-p)$ or $t > t_{\alpha/2}(n-p)$

   (2) Complete the test scheme using p-value.

   $H_0 : l'\beta = b$ vs $H_a : l'\beta \neq b$
   Test Statistic: $t = \frac{l'\widehat{\beta} - b}{S_{l'\widehat{\beta}}}$
   p-value: $2P(t(n-p) > |t_{ob}|)$

   (3) Show that α-level rejects H_0 if and only if p-value$< \alpha$.

$$
\begin{aligned}
\alpha\text{-level test rejects } H_0 &\iff t_{ob} < -t_{\alpha/2}(n-p) \text{ or } t_{ob} > t_{\alpha/2}(n-p) \\
&\iff |t_{ob}| > t_{\alpha/2}(n-p) \\
&\iff P(t(n-p) > |t_{ob}|) < P(t(n-p) > t_{\alpha/2}(n-p)) = \tfrac{\alpha}{2} \\
&\iff 2P(t(n-p) > |t_{ob}|) < \alpha \\
&\iff p\text{-value} < \alpha.
\end{aligned}
$$

2. Consider upper-sided alternative t-test

   (1) Complete the α-level test scheme.

   $H_0 : l'\beta \leq b$ vs $H_a : l'\beta > b$
   Test Statistic: $t = \frac{l'\widehat{\beta} - b}{S_{l'\widehat{\beta}}}$
   Reject H_0 if $t > t_\alpha(n-p)$

   (2) Show that the level of the test in (1) is α.

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\begin{aligned}
P(\text{Rejecting } H_0 | H_0 \text{ is true}) &= P(t > t_\alpha(n-p) | H_0 \text{ is true}) \\
&= P\left(\frac{l'\widehat{\beta} - b}{S_{l'\widehat{\beta}}} > t_\alpha(n-p) | H_0 \text{ is true}\right) \\
&= P\left(\frac{l'\widehat{\beta} - l'\beta}{S_{l'\widehat{\beta}}} - \frac{b - l'\beta}{S_{l'\widehat{\beta}}} > t_\alpha(n-p) | H_0 \text{ is true}\right) \\
&= P\left(t(n-p) - \frac{b - l'\beta}{S_{l'\widehat{\beta}}} > t_\alpha(n-p) | H_0 \text{ is true}\right) \\
&= P\left(t(n-p) > t_\alpha(n-p) + \frac{b - l'\beta}{S_{l'\widehat{\beta}}} | l'\beta \leq b\right) \\
&\leq P(t(n-p) > t_\alpha(n-p)) = \alpha.
\end{aligned}
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3. One-way ANOVA data with 4 levels: a, b, c, d are stored in mydata.dat. Find 90% CI for μ_d.
   Write formula, plug in numbers, present the final result. Use SAS.

$\overline{X}_d \pm t_{\alpha/2}(n-p)S_{\overline{X}_d} = \overline{X}_d \pm t_{0.05}(18)\sqrt{\frac{MSE}{n_d}} = 40 \pm 1.7341\,\sqrt{\frac{11.47407}{6}} = (37.602,\, 42.398)$
is a 90% confidence interval for μ_d.