Stat873　　　　　HW03

1. For $A' = A \in R^{n\times n}$ and $B' = B \in R^{n\times n}$ prove the following two statements

   (1) If $A \geq 0$, then $TAT' \geq 0$ for all $T \in R^{m\times n}$.
   Hint: Give the definition for $A \geq 0$ first.

   Note that $A \geq 0 \overset{def}{\Longleftrightarrow} x'Ax \geq 0$ for all $x \in R^n$.
   For all $x \in R^m$, with $y = T'x \in R^n$, by $A \geq 0$, $x'TAT'x = y'Ay \geq 0$.
   By the definition, $TAT' \geq 0$.

   (2) If $A \geq B$, then $TAT' \geq TBT'$ for all $T \in R^{m\times n}$.
   Hint: Give the definition of $A \geq B$ first.

   Note that $A \geq B \overset{def}{\Longleftrightarrow} A - B \geq 0$, i.e., $x'(A-B)x \geq 0$ for all $x \in R^n$.
   For $x \in R^m$, with $y = T'x \in R^n$, by $A \geq B$, $x'T(A-B)T'x = y'(A-B)y \geq 0$.
   Thus $T(A-B)T' = TAT' - TBT' \geq 0$.
   Hence $TAT' \geq TBT'$.

2. Prove the following statements

   (1) If $\widehat{\eta}$ is a LUE for η, then $A\widehat{\eta}$ is a LUE for $A\eta$.

   If $\widehat{\eta}$ is a LUE for η, then $\widehat{\eta} = LY$ for some L is a linear function of Y, and $E(\widehat{\eta}) \equiv \eta$. So $A\widehat{\eta} = ALY$ is a linear function of Y, and $E(ALY) = AE(LY) = A\eta$.
   Hence $A\widehat{\eta}$ is a LUE for $A\eta$.

   (2) If $\widehat{\eta}$ is a BLUE for η and B is non-singular, then $B\widehat{\eta}$ is a BLUE for $B\eta$.

   $\widehat{\eta}$ is a BLUE for $\eta \Longrightarrow \widehat{\eta}$ is a LUE for η.　　By (1) in 2,

   $$B\widehat{\eta} \in \text{LUE}(B\eta).$$

   Suppose $\widehat{\xi} \in \text{LUE}(B\eta)$, we need to show

   $$r(B\widehat{\eta},\, B\eta) = \text{Cov}(B\widehat{\eta}) \leq \text{Cov}(\widehat{\xi}) = r(\widehat{\xi},\, B\eta).$$

   $\widehat{\xi} \in \text{LUE}(B\eta)$.　　By (1) in 2,　　$B^{-1}\widehat{\xi} \in \text{LUE}(B^{-1}B\eta) = \text{LUE}(\eta)$.
   But $\widehat{\eta}$ is the BLUE for η. So

   $$\text{Cov}(\widehat{\eta}) \leq \text{Cov}(B^{-1}\widehat{\xi}).$$

   By (2) of 1,　　$B\left[\text{Cov}(\widehat{\eta})\right]B' \leq B\left[\text{Cov}(B^{-1}\widehat{\xi})\right]B'$, i.e.,

   $$\text{Cov}(B\widehat{\eta}) \leq \text{Cov}(\widehat{\xi}).$$