Stat873 **HW08**

1. In one-way ANOVA $Y = J\mu + \epsilon$, $Y \in R^n$, $J = \begin{pmatrix} 1_{n_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1_{n_p} \end{pmatrix} \in R^{n\times p}$, $\mu = \begin{pmatrix} \mu_1 \\ \vdots \\ \mu_p \end{pmatrix} \in R^p$ and $\epsilon \sim N(0,\, \sigma^2 I_n)$.

   (1) Show that $\mathrm{R}(1_n) \subset \mathcal{R}(J)$.

   (2) Find H_0 under which the model is reduced to $Y = 1_n\gamma + \epsilon$, $\epsilon \sim N(0,\, \sigma^2 I_n)$.

2. In 1 $Y = \begin{pmatrix} Y_1 \\ \vdots \\ Y_p \end{pmatrix} \in R^n$ where $Y_i = \begin{pmatrix} y_{i1} \\ \vdots \\ y_{in_i} \end{pmatrix} \in R^{n_i}$, $i = 1, ..., p$. Let $n_1 + \cdots + n_p = n$, $\overline{y}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} y_{ij}$, $\overline{y} = \frac{1}{n}\left(n_1\overline{y}_1 + \cdots + n_p\overline{y}_p\right)$ and $\mathrm{CSS}_i = \sum_{j=1}^{n_i}(y_{ij} - \overline{y}_i)^2$, $i = 1, ..., p$. Express the followings only using y_{ij}, $\overline{y}_i$, $\overline{y}$, CSS_i and summations.

   (1) SSE

   (2) C.SSTO

   (3) SSM

3. Data file T6-10.dat contains four variables y, x_1, x_2 and Type. Consider regression model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$, $\epsilon \sim N(0,\, \sigma^2)$.

   (1) With $x_1 = 1$ and $x_2 = -1$, find a 90% confidence interval for $E(y)$.

   (2) With $x_1 = 0.5$ and $x_2 = -0.5$, find a 90% prediction interval for y.

   (3) Report your test on $H_0 : 2\beta_0 - \beta_2 = 8$.