

For  $Y = X\beta + \epsilon$ ,  $\epsilon \sim N(0, \sigma^2\Sigma)$ ,  $X \in R^{n \times p}$  has full column rank.  $\theta = l'\beta \in R$  has BLUE  $l'\hat{\beta}$  where  $\hat{\beta}$  is the minimum norm GLSE $_{\Sigma^{-1}}(\beta)$ .

1. Find  $\sigma_{l'\hat{\beta}}^2$ , the variance of  $l'\hat{\beta}$ .
2. Replacing parameters in the expression of  $\sigma_{l'\hat{\beta}}^2$  by their UEs one can get the estimated the variance of  $l'\hat{\beta}$ ,  $S_{l'\hat{\beta}}^2$ . Find  $S_{l'\hat{\beta}}^2$ .
3. It is known that  $F(1, n-p) = [t(n-p)]^2$ . Derive the relation of  $F_{\alpha}(1, n-p)$  and  $t_{\alpha/2}(n-p)$ .
4. Express the  $1 - \alpha$  confidence interval for  $\theta = l'\beta$  derived in the lecture using the cut-off point for  $t(n-p)$  distribution and  $S_{l'\hat{\beta}}$ .