**Stat873** **HW05**

1. For $\beta \in R^p$ in $Y = X\beta + \epsilon$, $\epsilon \sim (0,\, \sigma^2 I_n)$, with full column rank $X \in R^{n\times p}$, $\widehat{\beta}$ is the BLUE and $\widehat{\beta}(q)$ is a principal component estimator.

   (1) Is $\widehat{\beta}(q)$ a linear biased estimator? Why?

   (2) Show that $\|\widehat{\beta}(q)\|^2 \le \|\widehat{\beta}\|^2$.

2. In order to find principal component estimator for $\beta$ in $Y = X\beta + \epsilon$, $\epsilon \sim (0,\, \sigma^2\Sigma)$, convert the model equivalently to $Y_* = X_*\beta + \epsilon_*$, $\epsilon_* \sim (0,\, \sigma^2 I_n)$.

   (1) Point out the relations of $Y_*$ and $Y$; $X_*$ and $X$; and $\epsilon_*$ and $\epsilon$.

   (2) Based on the second model, results in the lecture, and relations in (1), describe the principal component estimator for $\beta$ in the first model.

3. Let $\widehat{\beta}_1$ be the BLUE of $\beta$ in $Y_1 = X_1\beta + \epsilon_1$, $\epsilon_1 \sim (0,\, \sigma^2 I_n)$; and $\widehat{\beta}_2$ be the BLUE of $\beta$ in $Y_2 = X_2\beta + \epsilon_2$, $\epsilon_2 \sim (0,\, \sigma^2\Sigma)$. Write the mixed BLUE of $\beta$ as weighted average of $\widehat{\beta}_1$ and $\widehat{\beta}_2$ and point out the matrix weights.