Stat873 **HW04**

1. Consider Model $Y = X\beta + \epsilon$, $\epsilon \sim N(0, \sigma^2\Sigma)$.
   (1) Among all maximum likelihood estimators for β, point out the one with minimum norm.
   (2) Which norm was used in (1)? Why not $\|\cdot\|_{\Sigma^{-1}}$?
   (3) Suppose X has full column rank. Find the distribution for the estimator in (1).

2. In Model $Y = X\beta + \epsilon$, $\epsilon \sim N(0, \sigma^2 I_n)$, X has full column rank, and $X'X = P\Lambda P'$ is the EVD.
   (1) Let $\widehat{\beta}$ be the MVUE for β. Write out the expression for $\widehat{\beta}$ and its distribution.
   (2) Let $\widehat{\beta}(K) = [P(\Lambda + K)P']^{-1}X'Y$ be the ridge estimator for β. Express matrix A via P, Λ and K such that $\widehat{\beta}(K) = A\widehat{\beta}$.
   (3) Find the expression for $\mathrm{Cov}(\widehat{\beta}(K))$ via σ^2, P, Λ and K only.
   Hint: $\mathrm{Cov}(\widehat{\beta}(K)) = A[\mathrm{Cov}(\widehat{\beta})]A'$.
   (4) Based on (3) find $\mathrm{tr}[\mathrm{Cov}(\widehat{\beta}(K))]$ via σ^2, Λ and K only.