Stat873 HW02

1. With $E(Y_f) = H\beta \in R^q$, $\text{LUP}(Y_f) = \text{LUE}(H\beta) = \left[H(U^{1/2}X)^+U^{1/2} + \mathcal{N}(I_q,\, X)\right] Y$.
   Here $\mathcal{A} = H(U^{1/2}X)^+U^{1/2} + \mathcal{N}(I_q,\, X)$ is an affine set in $R^{q\times n}$.

   (1) Show that $H(U^{1/2}X)^+U^{1/2}XX^+ \in \mathcal{A}$.
   Comment: Consequently $\mathcal{A} = H(U^{1/2}X)^+U^{1/2}XX^+ + \mathcal{N}(I_q,\, X)$.

   (2) Show that $H(U^{1/2}X)^+U^{1/2}XX^+ \perp \mathcal{N}(I_q,\, X)$.
   Comment: Consequently, in $\mathcal{A}$, $H(U^{1/2}X)^+U^{1/2}XX^+$ has minimum norm.

2. $\text{MSE}(\widehat{u},\, v) = E\|\widehat{u}-v\|^2 = E[(\widehat{u}-v)'(\widehat{u}-v)]$ is a real-valued risk when v is predicted/estimated by $\widehat{u} \in R^q$. The matrix-valued risk in the lecture is denoted as $\text{MSEM}(\widehat{u},\, v)$.

   (1) Suppose $q = 1$. Show that $\text{MSEM}(\widehat{u},\, v) = \text{MSE}(\widehat{u},\, v)$.

   (2) Show that $\text{MSE}(\widehat{u},\, v) = \text{tr}\,[\text{MSEM}(\widehat{u},\, v)]$.
   Hint: $E[\text{tr}(X)] = \text{tr}[E(X)]$.

   (3) Show that if $\widehat{u}$ dominates $\widetilde{u}$ bu $\text{MSEM}(\cdot,\cdot)$, then $\widehat{u}$ dominates $\widetilde{u}$ by $\text{MSE}(\cdot,\cdot)$.
   Hint: $A \le 0 \Longrightarrow \text{tr}(A) \le 0$, and $A \ge 0 \Longrightarrow \text{tr}(A) \ge 0$.