Stat873 HW01

1. $\mathcal{A}$ is an affine set in linear space V. Then $0 \in \mathcal{A} \Longleftrightarrow \mathcal{A}$ is a subspace.
   Show $\Rightarrow$ only.
   Hint: For x, $y \in \mathcal{A}$, one needs to show $\alpha x + \beta y \in \mathcal{A}$ for all scalars α and β.
   Discuss three cases: (i) $\alpha + \beta \neq 0$, (ii) $\alpha + \beta = 0$ and $\alpha = 0$, (iii) $\alpha + \beta = 0$, but $\alpha \neq 0$.

2. In linear space V,

   $$\mathcal{A} \text{ is an affine set} \Longleftrightarrow \mathcal{A} = x_0 + S \text{ where } x_0 \in V \text{ and } S \text{ is a subspace in } V.$$

   Show $\Rightarrow$ only.
   Hint: Take $x_0 \in \mathcal{A}$. Then $\mathcal{A} = x_0 + \mathcal{A} - x_0$. Using 1 to show $\mathcal{A} - x_0$ is a space.

3. $\mathcal{A} = x_0 + S$ is an affine set in V where S is a subspace. If $x_1 \in \mathcal{A}$, then $\mathcal{A} = x_1 + S$.
   Show $\mathcal{A} \supset x_1 + S$ only.

4. For affine set $\mathcal{A} = x_0 + S$ in 3, $x_1 = x_0 - \pi(x_0|S) \in x_0 + S = \mathcal{A}$. So $\mathcal{A} = x_1 + S$.
   Show that among all $x \in \mathcal{A}$, $\|x_1\|^2 \leq \|x\|^2$.