Stat873 **Exam 3** **April 10, 2024**

Name:

1. For Model $Y \sim N(X\beta, \sigma^2\Sigma)$ where $X \in R^{n\times p}$ with $\text{rank}(X) = p$ and $1_n \notin \mathcal{R}(X)$, $R^2 = \frac{SSM}{SSTO}$, called the coefficient of determination, is the proportion of the variation in Y explained by the model.

   (1) Define F in ANOVA table via R^2. (10 points)

   (2) Replacing SSE and SSTO in $R^2 = \frac{SSM}{SSTO} = \frac{SSTO-SSE}{SSTO}$ by MSE and MSTO we obtain R^2_{adj}, the adjusted R^2. Express F via R^2_{adj}. (20 points)

   (3) R^2_{adj} could assume negative values. Find the condition on R^2 for $R^2_{adj} < 0$. (20 points)

2. Model M: $Y \sim N(X\beta, \sigma^2 I_n)$ where $X = (X_I, X_{II}) \in R^{n\times p}$ has full column rank and $X_I \in R^{n\times p_1}$. With $\beta = \begin{pmatrix}\beta_I \\ \beta_{II}\end{pmatrix} \in R^p$ where $\beta_I \in R^{p_1}$, $H_0 : \beta_{II} = 0$ reduces M to Model M_*. Let SSE and SSE_* be from M and M_*. Define $SSD=SSE_*-SSE$.

(1) Fill out the form below. Write SS as quadratic forms. (15 points)

Source	SS	DF
Difference	SSD=____________________	__________
Error M	SSE=____________________	__________
Error M_*	SSE_*=____________________	__________

(2) Derive the distribution of $\frac{SSD}{\sigma^2}$ under H_0. (20 points)

(3) Show that SSD and SSE are independent. (15 points)