Stat873 **Exam 2** **Mar. 6, 2024**

Name:

1. In model $Y = X\beta + \epsilon$, $\epsilon \sim N(0, \sigma^2\Sigma)$, $X \in R^{n\times p}$ has full column rank.
   (1) Write out $\widehat{\beta}$, the MVUE for β, and its distribution. (5 points)
   (2) Write out the two risks, $\text{MSEM}(\widehat{\beta}, \beta)$ and $\text{MSE}(\widehat{\beta}, \beta)$. (10 points)

2. Let $\widehat{\beta}(s) = s\widehat{\beta}$ where $0 < s < 1$ and $\widehat{\beta}$ is in 1 (1).
   (1) Show that $\widehat{\beta}(s)$ is a linear biased estimator for β. (10 points)
   (2) Find the risk $\text{MSE}(\widehat{\beta}(s), \beta)$. (10 points)
   (3) Find a sufficient and necessary condition on s for $\widehat{\beta}(s)$ to dominate $\widehat{\beta}$ by $\text{MSE}(\cdot, \cdot)$. Hint: Write $\text{MSE}(\widehat{\beta}(s), \beta) - \text{MSE}(\widehat{\beta}, \beta)$ as a function of s. (20 points)

3. Consider Bayesian approach for estimating β in 1.
   (1) Find likelihood function $L(\beta)$ and $f_1(\beta)$ such that $L(\beta) \propto f_1(\beta)$. (10 points)
   (2) With prior $\beta \sim N(\beta_0, \Sigma_0)$, its pdf $f_\beta(\beta) \propto f_0(\beta)$. This $f_0(\beta)$ was given in the lecture. Based on the relation $f_{\beta|Y}(\beta) \propto f_0(\beta) f_1(\beta)$, find the posterior distribution of β given Y. (20 points)
   (3) Find the Bayesian estimator $\widehat{\beta}_B$ for β and express it as a weighted average of β_0 and $\widehat{\beta}$ in 1. (15 points)