Stat873 **Exam 1** **Feb. 7, 2024**

Name:

1. For $Y = X\beta + \epsilon$ with $\epsilon \sim (0, \sigma^2\Sigma)$ under $G\beta = b$, the collection of all restricted generalized least square estimators for β is

$$\text{RGLSE}_{\Sigma^{-1}}(\beta) = \widehat{\beta} + (I - G^+G)\mathcal{N}\left(X(I - G^+G)\right)$$

where $\widehat{\beta} = G^+b + \left[\Sigma^{-1/2}X(I - G^+G)\right]^+ \Sigma^{-1/2}(Y - XG^+b)$.
Show that $\widehat{\beta} \in \text{RGLSE}_{\Sigma^{-1}}(\beta)$, and has minimum norm among all estimators in $\text{RGLSE}_{\Sigma^{-1}}(\beta)$.
(25 points)

2. If $LY + d$ is P1UE for $H\beta$ under $G\beta = b$ in Model A: $Y = X\beta + \epsilon$, $\epsilon \sim (0, \sigma^2\Sigma)$, then LY_* is a LUE for $H(I - G^+G)\eta$ in Model B: $Y_* = X_*\eta + \epsilon$, $\epsilon \sim (0, \sigma^2\Sigma)$ and $X_* = X(I - G^+G)$. Prove this statement. (25 points)

3. For Model A: $Y = X\beta + \epsilon$, $\epsilon \sim (0, \sigma^2\Sigma)$, under $G\beta = b$, $\beta = G^+b + (I - G^+G)\eta$. So $H\beta = HG^+b + H(I - G^+G)\eta$.
Let $\widehat{\xi}$ be the BLUE for $H(I - G^+G)\eta$ in Model B: $Y_* = X_*\eta + \epsilon$, $\epsilon \sim (0, \sigma^2\Sigma)$.
With $Y_* = Y - XG^+b$ and $X_* = X(I - G^+G)$, show that $HG^+b + \widehat{\xi}$ is the best PIUE for $H\beta$ under $G\beta = b$ in Model A. (25 points)

4. Present the followings (25 points)

(1) For Model $Y = X\beta + \epsilon$, $\epsilon \sim (0, \sigma^2\Sigma)$, write down the best PIUE for $H\beta$ under the restriction $G\beta = b$.

(2) For Model $Y = X\beta + \epsilon$, $\epsilon \sim (0, \sigma^2 I_n)$, by (1) write down the best PIUE for $H\beta$ under the restriction $G\beta = b$.

(3) For Model $Y = X\beta + \epsilon$, $\epsilon \sim (0, \sigma^2\Sigma)$, by (1) write down the BLUE for $H\beta$ under $G\beta = 0$.

(4) For Model $Y = X\beta + \epsilon$, $\epsilon \sim (0, \sigma^2 I)$, by (1) write down the BLUE for $H\beta$ under $G\beta = 0$.

(5) For Model $Y = X\beta + \epsilon$, $\epsilon \sim (0, \sigma^2\Sigma)$, by (1) write down the BLUE for $H\beta$.

(6) For Model $Y = X\beta + \epsilon$, $\epsilon \sim (0, \sigma^2 I_n)$, by (1) write down the BLUE for $H\beta$.