Stat776 HW10

1. Let $H = X'(XX')^{-1}X$ be the hat-matrix where $X \in R^{q\times n}$ has full row rank.

   (1) Simplify $XH, X(I-H)$ and $H'H$.

   $XH = X[X'(XX')^{-1}X] = X$
   $X(I-H) = X - X = 0.$
   $H'H = [X'(XX')X]'[X'(XX')^{-1}X] = X'(XX')^{-1}X = H.$

   (2) Find $\mathrm{tr}(I-H)$.

   $\mathrm{tr}(I-H) = \mathrm{tr}[I_n - X'(XX')^{-1}X] = \mathrm{tr}(I_n) - \mathrm{tr}[XX'(XX')^{-1}] = n - \mathrm{tr}(I_q) = n - q$

2. Consider regression $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \beta \begin{pmatrix} 1 \\ x \end{pmatrix} + \epsilon$, $\epsilon \sim N(0, \Sigma)$ with observed $Y = \begin{pmatrix} 1 & 8 & 4 & 2 \\ 7 & 4 & 6 & 2 \end{pmatrix}$ and $x = \begin{pmatrix} 1 & 2 & 4 & 2 \end{pmatrix}$.

   (1) Calculate via SAS (Keep 4 digits after decimal point)

   (i) $\widehat{\beta}$, the least square estimator for parameter matrix β

   (ii) the residual matrix $Y - \widehat{Y}$

   (iii) the estimated mean of y when $x = 3$

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data a;
  input y1 y2 x @@;
  datalines;
  1 7 1 8 4 2 4 6 4 2 2 2 . . 3
  ;
proc reg;
  model y1 y2=x/p;
  run;
```

   (i) $\widehat{\beta} = \begin{pmatrix} 2.2015 & 0.6842 \\ 4.6316 & 0.0526 \end{pmatrix}$

   (ii) $Y - \widehat{Y} = \begin{pmatrix} -1.8974 & 4.4211 & -0.9474 & -1.5789 \\ 2.3158 & -0.7368 & 1.1579 & -2.7368 \end{pmatrix}$

   (iii) $\widehat{y}(3) = \begin{pmatrix} 4.2632 \\ 4.7895 \end{pmatrix}$

   (2) Based on the result in (1) calculate
   (i) the error matrix E (ii) the unbiased estimator for Σ.

   (i) $E = (Y - \widehat{Y})(Y - \widehat{Y})' = \begin{pmatrix} 26.5367 & -4.4273 \\ -4.4273 & 14.7366 \end{pmatrix}$

   (ii) $S = \frac{E}{n-q} = \frac{E}{2} = \begin{pmatrix} 13.2684 & -2.2137 \\ -2.2137 & 7.3683 \end{pmatrix}$