Stat776 HW08

1. For two-sample T^2-test it is known that $T^2_{ob} = \left(\frac{1}{\Lambda} - 1\right)(n-2)$.

   (1) Let r be the Roy's greatest root. Derive a formula for computing T^2_{ob} based on r.

   It has been derived in class that $\Lambda = \frac{1}{1+r}$.
   So $T^2_{ob} = \left(\frac{1}{\Lambda} - 1\right)(n-2) = (1 + r - 1)(n-2) = (n-2)r$.

   (2) Let Pt be the Pillai's trace. Derive a formula for computing T^2_{ob} based on Pt.

   It has been derived in class that $Pt = \frac{r}{1+r}$. So $r = \frac{Pt}{1-Pt}$.
   Hence $T^2 = (n-2)r = \frac{(n-2)\,Pt}{1-Pt}$.

2. File T6-10.dat contains four variables x1, x2, x3 and type= $\begin{cases} \text{diesel} \\ \text{gasoline} \end{cases}$ that are forms two samples from $X_d \sim N(\mu_d, \Sigma)$ and $X_g \sim N(\mu_g, \Sigma)$. Here $\mu_d = \begin{pmatrix} \mu_{d1} \\ \mu_{d2} \\ \mu_{d3} \end{pmatrix}$ and $\mu_g = \begin{pmatrix} \mu_{g1} \\ \mu_{g2} \\ \mu_{g3} \end{pmatrix}$.

   (1) Report your test on $H_0 : \mu_d - \mu_g = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$.

```
data a;
   infile "D:\T6-10.dat";
   input x1 x2 x3 type $;
   if type='gasoline' then do;
       x1=x1-2;
       x2=x2+1;
       x3=x3+1;
   end;
proc anova;
   class type;
   model x1 x2 x3=type/nouni;
   manova h=type;
   run;
```

$H_0 : \mu_d - \mu_g = \delta_0$ vs $H_a : \mu_d - \mu_g \neq \delta_0$ where $\delta_0 = (-2, 1, 1)'$

Test statistic: $T^2 = (\overline{X}_d - \overline{X}_g - \delta_0)' \left(\frac{n}{n_1 n_2} S_p\right)^{-1} (\overline{X}_d - \overline{X}_g - \delta_0)$

P-value: $P(T^2(3,\, n-2) > T^2_{ob})$

$T^2_{ob} = \left(\frac{1}{\Lambda} - 1\right)(n-2) = \left(\frac{1}{0.6467} - 1\right) \times 57 = 31.140$

p-value: $P(T^2(3,\, 57) > T^2_{ob}) = P(F(3, 55) > 10.02) < 0.0001$

Reject H_0

(2) Report your test on $H_0: (\mu_{d1}+\mu_{d2})-(\mu_{g1}+\mu_{g2})=0$ and $(\mu_{d2}+\mu_{d3})-(\mu_{g2}+\mu_{g3})=10$.

```
data a;
   infile "D:\T6-10.dat";
   input x1 x2 x3 type $;
   y1=x1+x2;
   y2=x2+x3;
   if type='gasoline' then y2=y2+10;

proc anova;
   class type;
   model y1 y2=type/nouni;
   manova h=type;
   run;
```

$H_0: L(\mu_d-\mu_g)=\delta_0$ vs $H_a: L(\mu_d-\mu_g)\neq\delta_0$ where $L=\begin{pmatrix}1 & 1 & 0\\ 0 & 1 & 1\end{pmatrix}$ and $\delta_0=\begin{pmatrix}0\\ 10\end{pmatrix}$

Test statistic: $T^2=[L(\overline{X}_d-\overline{X}_g)-\delta_0]'\left(\frac{n}{n_1n_2}LS_pL'\right)^{-1}[L(\overline{X}_d-\overline{X}_g)-\delta_0]$

P-value: $P(T^2(2,\, n-2)>T^2_{ob})$

$T^2_{ob}=\left(\frac{1}{\Lambda}-1\right)(n-2)=\left(\frac{1}{0.9933}-1\right)\times 57=0.3845$

p-value: $P(T^2(2,\, 57)>T^2_{ob})=P(F(2,56)>0.19)<0.8284$

Fail to reject H_0