Stat776 HW05

With data in 5.11 on page 263 from $N(\mu, \Sigma)$, SAS produced $n = 9$, $\overline{X} = \begin{pmatrix} 5.18556 \\ 16.0700 \end{pmatrix}$ and $S = \begin{pmatrix} 176.0042 & 287.2412 \\ 287.2412 & 527.8493 \end{pmatrix}$.

(1) Construct a 90% confidence region for $\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$ in the form $(\mu - b)' A (\mu - b) \le 1$. Keep 4 digits after decimal points. Caution: A but not A^{-1}.

$$(\mu - \overline{X})' \left(\tfrac{S}{n}\right)^{-1} (\mu - \overline{X}) < T^2_\alpha(p, n-1) = \tfrac{(n-1)p}{n-p} F_\alpha(p, n-p) = \tfrac{8\times 2}{7} F_{0.1}(2,7) = \tfrac{16}{7} \times 3.26 = 7.4514$$

$$9 \begin{pmatrix} \mu_1 - 5.1856 \\ \mu_2 - 16.0700 \end{pmatrix}' \begin{pmatrix} 176.0042 & 287.2412 \\ 287.2412 & 527.8493 \end{pmatrix}^{-1} \begin{pmatrix} \mu_1 - 5.1856 \\ \mu_2 - 16.0700 \end{pmatrix} \le 7.4514$$

$$\begin{pmatrix} \mu_1 - 5.1856 \\ \mu_2 - 16.0700 \end{pmatrix}' \begin{pmatrix} 0.0613 & -0.0334 \\ -0.0334 & 0.0204 \end{pmatrix} \begin{pmatrix} \mu_1 - 5.1856 \\ \mu_2 - 16.0700 \end{pmatrix} \le 1$$

is a 90% confidence region for $\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}$

(2) Find a 90% confidence interval for $\mu_1 - \mu_2$.

$$\begin{aligned} \overline{X}_1 - \overline{X}_2 \pm t_{\alpha/2}(n-1)\sqrt{\tfrac{(1,-1)S(1,-1)'}{n}} &= 5.18566 - 16.0700 \pm t_{0.05}(8)\sqrt{\tfrac{176.0042 + 527.8493 - 2\cdot 287.2412}{9}} \\ &= -10.8844 \pm 1.85938 \times 3.7914 = -10.8844 \pm 7.0496 \\ &= (-17.9340,\ -3.8348) \end{aligned}$$

is a 90% confidence interval for $\mu_1 - \mu_2$.

(3) Find simultaneous confidence intervals for μ_1 and μ_2 with overall confidence coefficient 90% by Bonferroni method.

$t_{\alpha/(2k)}(n-1) = t_{0.025}(8) = 2.30469$

$\mu_1 \in \overline{X}_1 \pm 2.30469 \times \sqrt{\frac{176.0042}{9}} = 5.18556 \pm 2.30469 \times 4.4222 = 5.1856 \pm 10.1918 = (-5.0062,\ 15.3774)$

and

$\mu_2 \in \overline{X}_2 \pm 2.30469 \times \sqrt{\frac{527.8493}{9}} = 16.0700 \pm 2.30469 \times 7.6583 = 16.0700 \pm 17.6501 = (-1.5801,\ 33.7201)$

are simultaneous confidence intervals with overall confidence coefficient 0.90%.

(4) Find simultaneous confidence intervals for μ_1 and μ_2 with overall confidence coefficient 90% by Scheffe method.

$T^2_\alpha(p,\ n-1) = T^2_{0.1}(2,\ 8) = \frac{16}{7} F_{0.1}(2,\ 7) = \frac{16}{7} \times 3.25744 = 2.7287^2$

$\mu_1 \in \overline{X}_1 \pm 2.7287 \times \sqrt{\frac{176.0042}{9}} = 5.18556 \pm 2.7287 \times 4.4222 = 5.1856 \pm 12.0667 = (-6.8811,\ 17.2523)$

and

$\mu_2 \in \overline{X}_2 \pm 2.7287 \times \sqrt{\frac{527.8493}{9}} = 16.0700 \pm 2.7287 \times 7.6583 = 16.0700 \pm 20.8972 = (-4.8272,\ 36.9672)$

are simultaneous confidence intervals with overall confidence coefficient 0.90%.