Stat 776 **Exam 3** **April 11, 2024**

Name:

1. By EVD of sample covariance matrix, for factor model, SAS produced the following output.

Factor Pattern		
		Factor 1
x1		0.96169
x2		-0.61893
x3		0.82126
Variance explained by each factor		
Factor	Weighted	Unweighted
Factor 1	8.72172	1.98239
Communality estimates		
Variable	Communality	Weight
x1	0.92485	6.000
x2	0.38307	3.000
x3	0.67447	3.000

(1) Write SAS proc code without data step that produced the above output. (10 points)

```
proc factor nfactor=1 cov;
     var x1 x2 x3;
     run;
```

(2) Find the five missing values in the output and fill them in the blanks. (25 points)
Keep 5 digits after decimal point

$\widehat{h}^2_{z1} = (\widehat{l}_{z1})^2 = 0.96169^2 = 0.92485$
$\widehat{h}^2_{z2} = (\widehat{l}_{z2})^2 = 0.61893^2 = 0.38307$
$\widehat{h}^2_{z3} = (\widehat{l}_{z3})^2 = 0.82126^2 = 0.67447.$
$\widehat{f}^2_{1z} = \sum_{i=1}^3 \widehat{h}^2_{zi} = 0.92485 + 0.38307 + 0.67447 = 1.98239.$
$\widehat{f}^2_1 = \sum_{i=1}^3 \widehat{h}^2_{zi} s_i^2 = 0.92485 \times 6 + 0.38307 \times 3 + 0.67447 \times 3 = 8.72172.$

(3) Find the largest eigenvalue of S, the estimated $\text{var}(\epsilon_{z1})$ and the estimated $\text{var}(\epsilon_2)$. (15 points)
Keep 5 digits after decimal point

$\lambda_1 = \widehat{f}^2_1 = 8.72172$
$\widehat{\psi}_{z1} = 1 - \widehat{h}^2_{x1} = 1 - 0.92485 = 0.07515$
$\widehat{\psi}_2 = s_2^2 - \widehat{h}^2_2 = 3 \times (1 - 0.38307) = 1.85079.$

2. A sample from $N(\mu_x, \Sigma)$ with id= 10 and a sample from $N(\mu_y, \Sigma)$ with $id = 20$ are stored in file exam3.txt by four variables x1, x2, x3 and id. We need to test

$$H_0 : L(\mu_x - \mu_y) = \delta_0 \text{ vs } H_a : L(\mu_x - \mu_y) \neq \delta_0 \text{ where } L = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & -2 \end{pmatrix} \text{ and } \delta_0 = \begin{pmatrix} 3 \\ -3 \end{pmatrix}.$$

(1) Write SAS code including data step for the test. (15 points)

```
data a;
   infile "D:\exam3.txt";
   input x1 x2 x3 id @@;
   y1=x1+x2;
   y2=x1-2*x3;
   if id=20 then do;
   y1=y1+3;
   y2=y2-3;
   end;
```

```
proc anova;
   class id;
   model y1 y2=id/nouni;
   manova h=id;
run;
```

(2) Find the five missing values in SAS output and fill them in blanks. (20 points)
Keep 5 digits after decimal point.

Statistic	Value	F-value	Num-DF	Den-DF	$Pr > F$
Wilks Lambda	0.90378	2.60837	2	49	0.0839
Pillai's trace	0.09622				
Hotelling-Lawley trace	0.10646				
Roy Greatest root	0.10646				

$49 = \text{Den-DF} = (n-2) - q + 1 = n - 2 - 2 + 1 = n - 3 \Longrightarrow n = 52.$
$T^2 = \left(\frac{1}{\Lambda} - 1\right)(n-2) = \left(\frac{1}{0.90378} - 1\right) \times 50 = 5.32320$
$T^2(2, n-2) = \frac{2(n-2)}{n-3} F(2, n-3) \Longrightarrow F = \frac{n-3}{2(n-2)} T^2 = \frac{49}{2\times 50} \times 5.32320 = 2.60837.$
Pillai's trace$= 1 - \Lambda = 1 - 0.90378 = 0.09622.$
$\Lambda = \frac{1}{1+r_1} \Longrightarrow r_1 = \frac{1}{\Lambda} - 1 = \frac{1}{0.90378} - 1 = 0.10646$
Hotelling-Lawley trace=Roy Greatest root$= r_1 = 0.10646.$

(3) Write a report on the test (15 points)

$H_0 : L(\mu_x - \mu_y) = \delta_0$ vs $H_a : L(\mu_x - \mu_y) \neq \delta_0$ where $L = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & -2 \end{pmatrix}$ and $\delta_0 = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$

Test statistic: $T^2 = [L(\overline{X} - \overline{Y}) - \delta_0]' \left(\frac{n}{n_1 n_2} L S_p L'\right)^{-1} [L(\overline{X} - \overline{Y}) - \delta_0].$
p-value: $P(T^2(2, n-2) > T^2_{ob}).$

$T^2_{ob} = 5.32320$
p-value: $P(T^2(2, 50) > 5.32320) = P(F(2, 49) > 2.60877) = 0.0839.$

Fail to reject H_0 at the level $\alpha = 0.08.$