Stat 776 Exam 2 Mar. 8, 2024

Name:

1. $X \sim N(2\mu, 3\Sigma)$ is independent to $W \sim W_{4\times 4}(7, 5\Sigma)$. Find a, b, c, k_1 and k_2 in

$$(X - a\mu)' \left(\frac{bW}{c}\right)^{-1} (X - a\mu) \sim T^2(k_1, k_2).$$

(20 points)

$X \sim N(2\mu, 3\Sigma) \Longrightarrow \frac{1}{\sqrt{3}}(X - 2\mu) \sim N(0, \Sigma)$. $W \sim W_{4\times 4}(7, 5\Sigma) \Longrightarrow \frac{1}{5}W \sim W_{4\times 4}(7, \Sigma)$.
X and W are independent$\Longrightarrow \frac{1}{\sqrt{3}}(X - 2\mu)$ and $\frac{1}{5}W$ are independent.
So $\left[\frac{1}{\sqrt{3}}(X - 2\mu)\right]' \left(\frac{W}{5\times 7}\right)^{-1} \left[\frac{1}{\sqrt{3}}(X - 2\mu)\right] = (X - 2\mu)' \left(\frac{3W}{35}\right)^{-1} (X - 2\mu) \sim T^2(4, 7)$.
Thus $a = 2$, $b = 3$ and $c = 35$, $k_1 = 4$ and $k_2 = 7$.

2. Are the following equations valid? (20 points)

(1) $W_{2\times 2}(5, I_2) = W_{2\times 2}(5)$	YES✓	NO
(2) $W_{1\times 1}(5) = \chi^2(5)$	YES✓	NO
(3) $T^2(1, 5) = T^2(5)$	YES	NO✓
(4) $T^2(5) = [t(5)]^2$	YES	NO✓
(5) $T^2(1, 5) = F(1, 5)$	YES✓	NO

3. In order to test $\mu = \begin{pmatrix} -5 \\ 10 \\ 6 \end{pmatrix}$ for μ in $N(\mu, \Sigma)$. Write SAS code assuming data are stored in a file with name test2.txt. (10 points)

```
data a;
    infile "D:\test2.tex";
    input x1 x2 x3 @ @;
    y1=x1+5;
    y2=x2-10;
    y3=x3-6;
proc reg;
    model y1 y2 y3=/noprint;
    mtext intercept;
    run;
```

4. SAS in 3 produced the output based which one wrote a report on the test.

Stat.	Value	F-value	N-DF	D-DF	Pr>F
Wilks' Lambda	0.7997	0.4174	3	5	0.7484
Pillai's trace	0.2003				
Hotelling-Layley trace	0.2504				
Roy's G-root	0.2504				

$H_0 : \mu = \mu_0$ vs $H_a : \mu \neq \mu_0$ where $\mu_0 = \begin{pmatrix} 5 \\ 10 \\ -6 \end{pmatrix}$

Test Statistic: $T^2 = (\overline{X} - \mu_0)' \left(\frac{S}{n}\right)^{-1} (\overline{X} - \mu_0)$

Reject H_0 if $T^2 > 22.72$ for $\alpha = 0.05$

$T^2_{ob} =$ 1.753

Conclusion: Fail to reject the null hypothesis

(1) Work out the missing values and complete the report. (30 points)

(1) $p = 3$
(2) $n - p = 5 \Longrightarrow n = 8.\ T^2 = \left(\frac{1}{\Lambda} - 1\right)(n-1) = 1.753$
(3) $T^2 = \frac{(n-1)p}{n-p} F \Longrightarrow F = \frac{n-p}{(n-1)p} T^2 = 0.4174$
(4) Pillai's trace$= 1 - \Lambda = 0.2003$
(5) Hotelling-Lawley trace=Roy's greatest root$= \frac{T^2}{n-1} = 0.2504$

(2) Is $\begin{pmatrix} -6 \\ 10 \\ 5 \end{pmatrix}$ in the 10% confidence region for μ? (5 points)

$1 - (p - \text{value}) = 1 - 0.7484 = 0.2516 > 0.10.$ No.

(3) Suppose $S = \begin{pmatrix} 2 & 1.5 & -1 \\ 1.5 & 4 & -0.5 \\ -1 & -0.5 & 6 \end{pmatrix}$. Among all Scheffe's intervals with overal confidence coefficient 95% find the width of the one for $\mu_3 - \mu_1$. (15 points)

With $l = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$, the width of the Scheffe's interval for $l'\mu$ is

$$\begin{aligned} W &= 2\sqrt{T^2_\alpha(p,\, n-1)\ S^2_{l'\overline{X}}} = 2\sqrt{T^2_\alpha(p,\, n-1)\ \ l'\frac{S}{n}l} \\ &= 2\sqrt{22.72 \times \frac{2+6-2\times(-1)}{8}} = 2\sqrt{22.72 \times \frac{10}{8}} = 2\sqrt{28.4} = 10.66 \end{aligned}$$