Stat776 **HW10**

1. Let $H = X'(XX')^{-1}X$ be the hat-matrix where $X \in R^{q\times n}$ has full row rank.

   (1) Simplify $XH, X(I-H)$ and $H'H$.

   (2) Find $\mathrm{tr}(I-H)$.

2. Consider regression $y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \beta \begin{pmatrix} 1 \\ x \end{pmatrix} + \epsilon,\ \epsilon \sim N(0, \Sigma)$ with observed $Y = \begin{pmatrix} 1 & 8 & 4 & 2 \\ 7 & 4 & 6 & 2 \end{pmatrix}$ and $x = \begin{pmatrix} 1 & 2 & 4 & 2 \end{pmatrix}$.

   (1) Calculate via SAS (Keep 4 digits after decimal point)

   (i) $\widehat{\beta}$, the least square estimator for parameter matrix β

   (ii) the residual matrix $Y - \widehat{Y}$

   (iii) the estimated mean of y when $x = 3$

   (2) Based on the result in (1) calculate

   (i) the error matrix E

   (ii) the unbiased estimator for Σ.