Stat776 HW09

1. The following is Factor model analysis table.

X_i	σ_i^2	F_1	F_2	h_i^2	ψ_i
X_1	19	16	1	17	2
X_2	57	49	4	53	4
X_3	38	1	36	37	1
X_4	68	1	64	65	3
	182	67	105	172	10

(1) Find the contribution of factor F to $\text{var}(X_2)$.

(2) Find the part of the total variance in X explained by F_2.

(3) Let Z be standardized X. Find the part of $\text{var}(Z_3) = 1$ contributed by F_2.

2. In 9.8 on page 531 $\text{Cov}(X)$, $\Sigma = \begin{pmatrix} 1 & 0.4 & 0.9 \\ 0.4 & 1 & 0.7 \\ 0.9 & 0.7 & 1 \end{pmatrix}$ is given.

Show that for this X there is no factor model $X - \mu = LF + \epsilon$ with $L \in R^{3\times 1}$.

3. Table 9-12 on page 536 is stored in T9-12.dat. Run

```
data a;
     infile "D:\T9-12.dat";
     input x1 x2 x3 x4 x5 x6 x7;
     run;
```

```
proc factor nfactor=2 cov;
    var x1 x2 x3;
    run;
```

Consider factor model $X - \mu = LF + \epsilon$ with $\epsilon \sim (0, \Psi)$ and factor model for standardized X, $Z = L_z F + \epsilon_z$ with $\epsilon_z \sim (0, \Psi_z)$. For the following computation problems keep 5 digits after decimal point for final results.

(1) Find $\widehat{\Psi}_z$, the estimated $\Psi_z = \text{Cov}(\epsilon_z)$.

(2) Find $\widehat{L}$, the estimated loading matrix L.

(3) Find $\widehat{\Psi}$, the estimated $\Psi = \text{Cov}(\epsilon)$.