Stat776 HW04

1. Suppose $X_1, ..., X_m$ is a random sample from $N(\mu_x, \Sigma)$ and $Y_1, ..., Y_n$ is a random sample from $N(\mu_y, \Sigma)$. Let $Z = (X_1, ..., X_m, Y_1, .., Y_n) \in R^{p\times(m+n)}$. Express the distribution of Z.

2. Find the following probabilities.

   (1) For $X \sim W_{1\times 1}(5)$, find $P(X > 12)$.

   (2) For $X \sim W_{1\times 1}(5, 4)$, find $P(X > 30)$.

   (3) $P(T^2(5, 14) > 3)$.

3. Let $\overline{X} \in R^4$ and $S \in R^{4\times 4}$ be from a sample of size 20 from $N(\mu, \Sigma)$.
   Define $Y = (\overline{X} - \mu)' \left(\frac{S}{20}\right)^{-1} (\overline{X} - \mu)$. Find $P(Y > 4)$.

4. Suppose $X \sim N_4(\mu, 4\Sigma)$ is independent to $W \sim W_{4\times 4}(16, \Sigma)$. In the following expression find a, b and c.

$$(X - \mu)' \left(\frac{W}{a}\right)^{-1} (X - \mu) \sim T^2(b, c)$$