Math 243: Calculus II
Exam 6: Ends of Chs 9 and 10

Due date: Wed, 5 Dec 2012

Name: _____

Instructions: Complete all problems, showing all work. Simplify as necessary. Leave any answers involving π or irreducible square roots in terms of such (no rounded off decimals).

1. Write a polar equation for an ellipse with one focus at the pole (origin), vertx at $(1, \pi/2)$, and eccentricity of 0.8.

2. Find the eccentricity, identify the conic, and give an equation of the directrix.

$$r = \frac{5}{2 - 2\sin\theta}$$

3. Find the area of the ellipse in problem 1. [You can use the Euclidean geometry formula $A = \pi ab$, or you can integrate.]

4. Find vector, parametric, and symmetric equations for the line through the origin and parallel to the line $x = 2t$, $y = 1 - t$, and $z = 4 + 3t$.

5. Find parametric equations for the line segment connecting $(10, 3, 1)$ to $(5, 6, -3)$ in this direction.

6. Determine whether the lines are parallel, skew, or intersecting. If intersecting, find the angle between them (in radians, to two decimal places).

$$\ell_1 : \quad \frac{x-1}{2} = \frac{y-3}{2} = \frac{z-2}{-1}$$

$$\ell_2 : \quad \frac{x-2}{1} = \frac{y-6}{-1} = \frac{z+2}{3}$$

7. Find an equation of the plane that contains the line $\vec{r}(t) = \langle 3+2t, t, 8-t \rangle$ and is parallel to the plane $2x + 4y + 8z = 17$.

8. Determine whether the planes are parallel, perpendicular, or neither. If neither, find the angle between them (in radians, to two decimal places).

$$2x - 3y + 4z = 5, \quad \text{and} \quad x + 6y + 4z = 3$$

9. Find the distance between the point $P(1, 0, -1)$ and the line

$$\vec{r}(t) = \langle 5 - t, 3t, 1 + 2t \rangle.$$

10. (a) Find the point where the given lines intersect, and (b) find an equation of the plane that contains them.

$$\vec{r}_1(t) = \langle 1, 1, 0 \rangle + t\langle 1, -1, 2 \rangle, \quad \text{and}$$
$$\vec{r}_2(t) = \langle 2, 0, 2 \rangle + s\langle -1, 1, 0 \rangle.$$

11. Show that the distance between parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is given by

$$D = \frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

12. Find the distance between the planes $3x + 6y - 9z = 4$ and $x + 2y - 3z = 1$.

13. What does $x^2 - y^2 = 1$ represent as a curve in $\mathbb{R}^2$? What does it represent as a surface in $\mathbb{R}^3$?

14. Find and identify the traces of the quadratic surface

$$x^2 + y^2 + 2y - z^2 = 0.$$

What does the graph look like in $\mathbb{R}^3$? Why?

15. Reduce the equation to one of the standard forms, then classify it.

$$4y^2 + z^2 - x - 16y - 4z + 20 = 0$$

16. Reduce the equation to one of the standard forms, then classify it.

$$x^2 - y^2 + z^2 - 2x + 2y + 4z + 2 = 0$$