Review Exercises : Exam 2

1. a) $f(x) = e^{2x^2+3x}$

$$f'(x) = (4x+3)\,e^{2x^2+3x}$$

b) $y = \dfrac{\ln(x)}{x+1}$, $\dfrac{dy}{dx} = \dfrac{(x+1)\frac{1}{x} - \ln(x)}{(x+1)^2} = \dfrac{1 + \frac{1}{x} - \ln(x)}{(x+1)^2}$

c) $g(x) = \ln(2x^2+4x)$

$$g'(x) = \frac{4x+4}{2x^2+4x} = \frac{\cancel{2}(2x+2)}{\cancel{2}(x^2+2x)} = \frac{2x+2}{x^2+2x}$$

d) $f(x) = x^3 e^x$

$$f'(x) = 3x^2 e^x + x^3 e^x = x^2 e^x (3+x)$$

e) $y = \ln(x^2)$

$$\frac{dy}{dx} = \frac{2x}{x^2} = \frac{2}{x} \quad \text{since } x \neq 0$$

OR $y = \ln(x^2) = 2\ln x \Rightarrow \dfrac{dy}{dx} = 2 \cdot \dfrac{1}{x} = \dfrac{2}{x}$

f) $f(x) = \dfrac{(x+2)^2}{x^2-4} = \dfrac{(x+2)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}} = \dfrac{x+2}{x-2}$, $x \neq -2$

$$f'(x) = \frac{(x-2)-(x+2)}{(x-2)^2} = \boxed{\frac{-4}{(x-2)^2}}, \boxed{x \neq -2}$$

1. g.) $h(x) = \log_7(3x)$

$$h'(x) = \frac{3}{3x\ln(7)} = \boxed{\frac{1}{x\ln(7)}}$$

h.) $y = 8^x$ $\quad \frac{dy}{dx} = \boxed{8^x \ln(8)}$

2. a.) Implicit: $xy + \ln(y) = y^2$

$$x\frac{dy}{dx} + y + \frac{1}{y}\frac{dy}{dx} = 2y\frac{dy}{dx}$$

$$\left(x + \frac{1}{y} - 2y\right)\frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = \frac{-y}{x + \frac{1}{y} - 2y} = \boxed{\frac{-y^2}{xy + 1 - 2y^2}}$$

b.) $x^2 + y^2 = 25$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y} = \boxed{\frac{-x}{y}}$$

c.) $e^{xy} = 3x + y$

$$e^{xy}\left(x\frac{dy}{dx} + y\right) = 3 + \frac{dy}{dx}$$

$$\left(xe^{xy} + 1\right)\frac{dy}{dx} = 3 - ye^{xy}$$

$$\frac{dy}{dx} = \boxed{\frac{3 - ye^{xy}}{xe^{xy} + 1}}$$

3. $\frac{dy}{dx} = \frac{-x}{y}$ from #2.b.

At $(8,-6)$, $\frac{dy}{dx} = \frac{-8}{-6} = \frac{4}{3} = m$

$$y - y_1 = m(x - x_1) \Rightarrow y + 6 = \frac{4}{3}(x-8)$$

$$y = \frac{4}{3}x - \frac{32}{3} - 6$$

$$\boxed{y = \frac{4}{3}x - \frac{50}{3}}$$

4. Check notes, quiz, etc.

5. a. $\lim\limits_{x\to\infty} \frac{x^2}{\ln x} = \frac{\infty}{\infty}$ L'H!

$$= \lim_{x\to\infty} \frac{2x}{\frac{1}{x}} = \lim_{x\to\infty} 2x^2 = \boxed{\infty}$$

b. $\lim\limits_{x\to 0^+} \frac{e^{1/x}}{\ln(x)} = \frac{e^{\infty}}{\ln 0} = \frac{\infty}{-\infty}$ L'H!

$$= \lim_{x\to 0^+} \frac{-\frac{1}{x^2}e^{1/x}}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{-e^{1/x}}{x^3} = \frac{-e^{\infty}}{0^+} = \boxed{-\infty}$$

c. $\lim\limits_{x\to 1} \frac{1-x}{\ln(x)} = \frac{1-1}{\ln(1)} = \frac{0}{0}$ = L'H!

$$= \lim_{x\to 1} \frac{-1}{\frac{1}{x}} = \lim_{x\to 1} -x = \boxed{-1}$$

6. $P = 2x + 2y = 225 \Rightarrow y = \dfrac{225 - 2x}{2}$

$$A = xy = x\left(\frac{225-2x}{2}\right) = \frac{225}{2}x - \frac{\cancel{2}x^2}{\cancel{2}}$$

$$A'(x) = \frac{225}{2} - 2x = 0$$

$$2x = \frac{225}{2}$$

$$\boxed{x = \frac{225}{4}} \Rightarrow \boxed{y = \frac{225}{4}}$$

7. See notes.

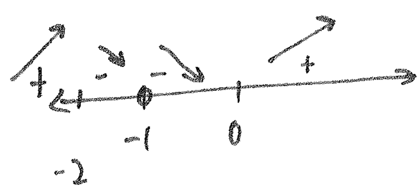
8. A. $f(x) = \dfrac{x^2}{x+1}$

a. $D_f = \{x \neq -1\}$

b. $f'(x) = \dfrac{(x+1)(2x) - x^2(1)}{(x+1)^2} = \dfrac{2x^2 + 2x - x^2}{(x+1)^2} = \dfrac{x^2+2x}{(x+1)^2}$

CVs: top: $x^2+2x = x(x+2) = 0$ bott: $x+1 = 0$

$x = 0,\ x = -2$ $x = -1$

sign chart: 

$$\boxed{\begin{array}{l} \text{Dec.}: (-2,-1) \cup (-1, 0) \\ \text{Inc.}: (-\infty, -2) \cup (0, \infty) \end{array}}$$

8. A. c. local max at $x=-2$, $f(-2)=\frac{(-2)^2}{-2+1}=\frac{4}{-1}=\boxed{-4}$

local min at $x=0$, $f(0)=\frac{0^2}{0+1}=\boxed{0}$

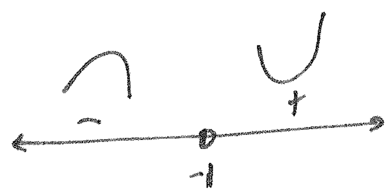
d. $f''(x)=\frac{(x+1)^2(2x+2)-(x^2+2x)(2)(x+1)}{(x+1)^4}=\frac{(x^2+2x+1)(2x+2)-(x^2+2x)(2x+2)}{(x+1)^4}$

$=\frac{\cancel{2x^3}+\cancel{4x^2}+2x+\cancel{2x^2}+\cancel{4x}+2-(\cancel{2x^3}+\cancel{4x^2}+\cancel{2x^2}+\cancel{4x})}{(x+1)^4}$

$=\frac{2x+2}{(x+1)^4}=\frac{2\cancel{(x+1)}}{(x+1)^{\cancel{4}\,3}}=\frac{2}{(x+1)^3}$

CVs of f': ~~top~~ bott. $x=-1$

sign chart:



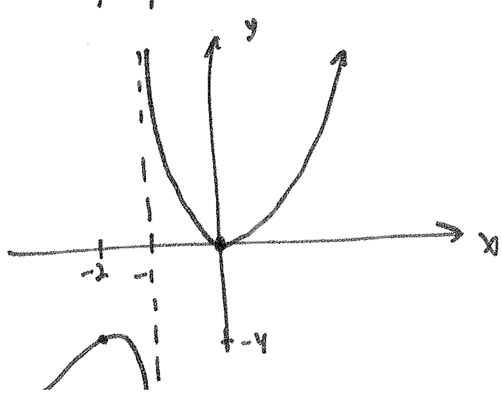
$\boxed{\begin{array}{l}\text{Conc. up: } (-1,\infty)\\ \text{conc. down: } (-\infty,-1)\end{array}}$

e. None since $x=-1 \notin D_f$.

f. V.A. at $x=-1$

No horizontal asymptotes

g.



8.B. $f(x) = x - \ln x$

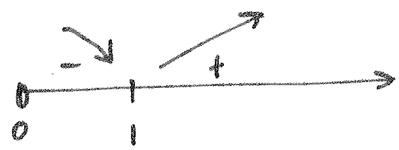
a. $D_f = \{x > 0\}$ because of log.

b. $f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x}$

CVs: top $x = 1$

bott none ($0 \notin D_f$).

Sign Chart:



$\boxed{\begin{array}{l} \text{Inc.: } (1, \infty) \\ \text{Dec.: } (0, 1) \end{array}}$

c. local min at $x = 1$, $f(1) = 1 - \ln(1) = 1 - 0 = \boxed{1}$

$(1, 1)$

d. $f''(x) = 0 + \frac{1}{x^2} = \frac{1}{x^2}$ always > 0.

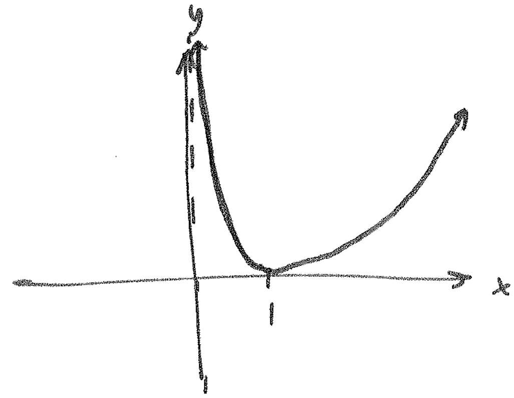
so f is $\boxed{\text{conc up on } (0, \infty)}$

e. None.

f. V.A. at $x = 0$

No H.A.

g.



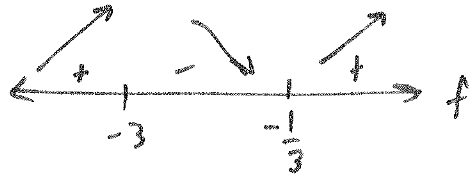
8.C. $f(x) = x^3 + 5x^2 + 3x + 1$

a. $D_f = \mathbb{R}$

b. $f'(x) = 3x^2 + 10x + 3 = 0$

$$x = \frac{-10 \pm \sqrt{100 - 36}}{6} = \frac{-10 \pm 8}{6} = \boxed{-3, \frac{-1}{3}} \text{ CVs}$$

Sign Chart:



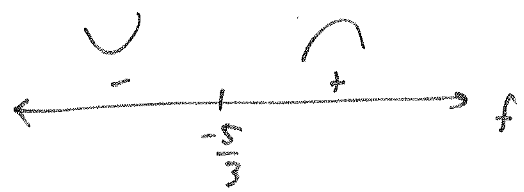
$$\boxed{\begin{array}{ll} \text{Inc:} & (-\infty, -3) \cup (-\tfrac{1}{3}, \infty) \\ \text{Dec:} & (-3, -1/3) \end{array}}$$

c. local max at $x = -3$, $f(-3) = -27 + 45 - 9 + 1 = 10$, $(-3, 10)$

local min at $x = -\frac{1}{3}$, $f(-\frac{1}{3}) = -\frac{1}{27} + \frac{5}{9} - 1 + 1 = \frac{14}{27}$, $(-\frac{1}{3}, \frac{14}{27})$

d. $f''(x) = 6x + 10 = 0$

$$x = \frac{-10}{6} = \frac{-5}{3}$$

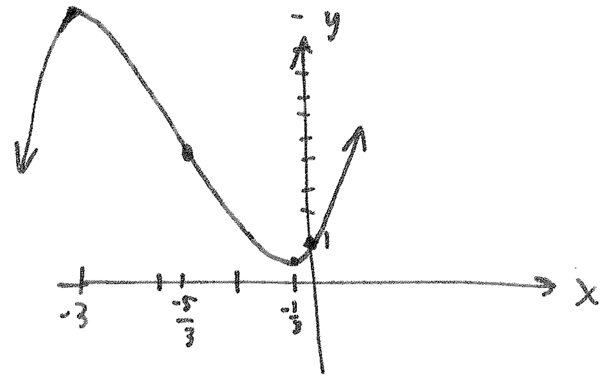


$$\boxed{\begin{array}{l} \text{C.Up: } (-\tfrac{5}{3}, \infty) \\ \text{C.Down: } (-\infty, \tfrac{-5}{3}) \end{array}}$$

e. Inf. Pt. $x = -\frac{5}{3}$, $f(-\frac{5}{3}) = \frac{142}{27} \approx 5.26$

8. C.f. None & None

9.



9. $y = x^4 + 3x^2 + 4$ on $[-1, 3]$

$$\frac{dy}{dx} = 4x^3 + 6x = 0$$

$$2x(2x^2 + 3) = 0$$

$x = 0$ $\quad x = \pm\sqrt{-3/2} = \underline{DNE}$

To check: $x = -1, 0, 3$

$f(-1) = 1 + 3 + 4 = 8$

$\boxed{f(0) = 4}$ ← Absolute min

$\boxed{f(3) = 81 + 27 + 4 = 112}$ ← Abs. max.