Review for Midterm Exam 1

Instructions. Here is a list of *some* of the things that you should know for the exam. Some are example problems that you should be able to work out. Others are terms or ideas that you should know. I won't ask you to write down any definitions or theorems word-for-word, but you should know what they say, what they mean, and how to use them.

1. **Completing the square.**

   - $f(x) = x^2 + 10x - 13$

   - $f(x) = 3x^2 + 3x - 6$

   - Solve: $x^2 - 5x + 1 = 0$

2. **Solving equations for x.**

   - $3^x = 27^{x-2}$

   - $\ln x = 3$

   - Solve: $e^{x+1} = 5$

3. **Limits using a graph.**

   - Check the class notes and homework.

4. **Continuity, Discontinuities, Domain.**

   - Find the domain. $f(x) = \dfrac{\sqrt{x-2}}{x^2 - x - 2}$

   - Intuitive definition of continuous at a point.

   - Limit definition of continuous at a point.

   - Continuity theorem for polynomials and rational functions.

   - Three types of discontinuities. When can each happen for a rational function?

   - Example: Quiz 3.

   - Determine when the function is (dis)continuous, and state what kind of discontinuity each one is:
     $$f(x) = \frac{x^2 - 20x + 19}{x^2 - 3x + 2}$$

5. **Algebraic Limits.**

- $\lim\limits_{x \to 3} 3x^2 - 7x + 1.$

- $\lim\limits_{x \to 5} \dfrac{(x-5)^2}{x^2 - 25}$

- $\lim\limits_{y \to -7^-} \sqrt{y+7}$

- $\lim\limits_{y \to -7^+} \sqrt{y+7}$

- $\lim\limits_{y \to -7} \sqrt{y+7}$

6. **Infinite Limits.**

- $\lim\limits_{x \to \infty} \dfrac{(x+3)(x-7)}{2x^2 + 5x - 9}$

- $\lim\limits_{x \to -4} \dfrac{x^2}{x^2 - 16}$

7. **Limits of piecewise functions.**

- Let $f(x) = \begin{cases} x - 2, & x < -2 \\ -x^2, & -2 \le x < 0 \\ \sqrt{x}, & x \ge 0 \end{cases}$

- Find $f(-2)$, $\lim\limits_{x \to -2} f(x)$, $f(0)$, $\lim\limits_{x \to 0} f(x)$. Is the function continuous at these points?

8. **Difference Quotients and Derivatives.**

- Find the difference quotients for $y = x^2$, $f(x) = \frac{1}{x}$, and $g(x) = \sqrt{x}$.

- Intuitive definition of derivative.

- Limit definition of derivative.

- Find the derivatives of the functions in the first bullet using the limit definition (no "tricks").

- Find an equation for the tangent line to $y = x^3 - 2x$ at $x = 1$.