Review Exercises

Work through all the problems in this chapter review and check your answers in the back of the book. Answers to all review problems are there along with section numbers in italics to indicate where each type of problem is discussed. Where weaknesses show up, review appropriate sections in the text.

A

In Problems 1–3, use point-by-point plotting to sketch the graph of each equation.

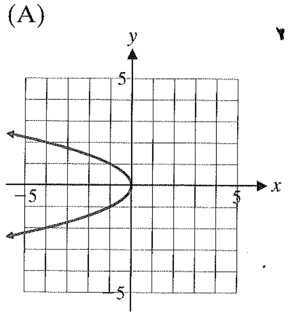
1. $y = 5 - x^2$

2. $x^2 = y^2$

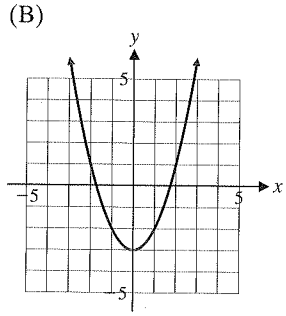
3. $y^2 = 4x^2$

4. Indicate whether each graph specifies a function:

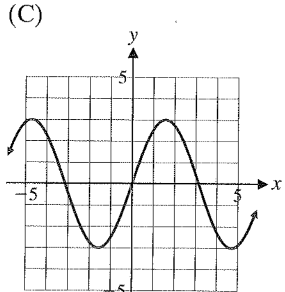
(A)



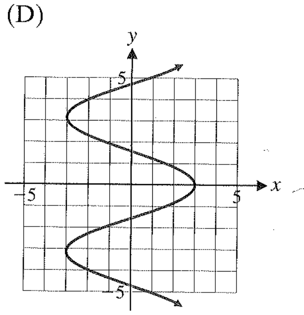
(B)



(C)



(D)



5. For $f(x) = 2x - 1$ and $g(x) = x^2 - 2x$, find:

(A) $f(-2) + g(-1)$ (B) $f(0) \cdot g(4)$

(C) $\dfrac{g(2)}{f(3)}$ (D) $\dfrac{f(3)}{g(2)}$

6. Write in logarithmic form using base e: $u = e^v$.

7. Write in logarithmic form using base 10: $x = 10^y$.

8. Write in exponential form using base e: $\ln M = N$.

9. Write in exponential form using base 10: $\log u = v$.

Solve Problems 10–12 for x exactly without using a calculator.

10. $\log_3 x = 2$ **11.** $\log_x 36 = 2$

12. $\log_2 16 = x$

Solve Problems 13–16 for x to three decimal places.

13. $10^x = 143.7$ **14.** $e^x = 503{,}000$

15. $\log x = 3.105$ **16.** $\ln x = -1.147$

17. Use the graph of function f in the figure to determine (to the nearest integer) x or y as indicated.

(A) $y = f(0)$ (B) $4 = f(x)$

(C) $y = f(3)$ (D) $3 = f(x)$

(E) $y = f(-6)$ (F) $-1 = f(x)$

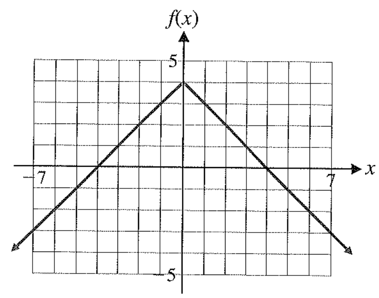


Figure for 17

18. Sketch a graph of each of the functions in parts (A)–(D) using the graph of function f in the figure below.

(A) $y = -f(x)$ (B) $y = f(x) + 4$

(C) $y = f(x - 2)$ (D) $y = -f(x + 3) - 3$

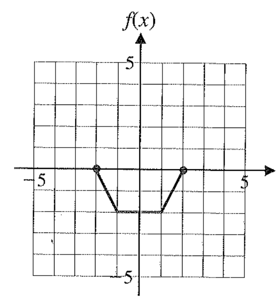


Figure for 18

19. Complete the square and find the standard form for the quadratic function

$$f(x) = -x^2 + 4x$$

Then write a brief verbal description of the relationship between the graph of f and the graph of $y = x^2$.

20. Match each equation with a graph of one of the functions f, g, m, or n in the figure.

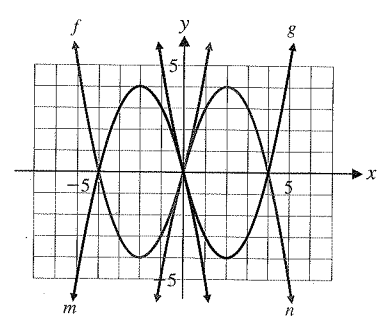


Figure for 20

(A) $y = (x - 2)^2 - 4$ (B) $y = -(x + 2)^2 + 4$

(C) $y = -(x - 2)^2 + 4$ (D) $y = (x + 2)^2 - 4$

21. Referring to the graph of function f in the figure for Problem 20 and using known properties of quadratic functions, find each of the following to the nearest integer:

(A) Intercepts (B) Vertex
(C) Maximum or minimum (D) Range

In Problems 22–25, each equation specifies a function. Determine whether the function is linear, quadratic, constant, or none of these.

22. $y = 4 - x + 3x^2$
23. $y = \frac{1 + 5x}{6}$
24. $y = \frac{7 - 4x}{2x}$
25. $y = 8x + 2(10 - 4x)$

Solve Problems 26–33 for x exactly without using a calculator.

26. $\log(x + 5) = \log(2x - 3)$
27. $2\ln(x - 1) = \ln(x^2 - 5)$
28. $9^{x-1} = 3^{1+x}$
29. $e^{2x} = e^{x^2-3}$
30. $2x^2e^x = 3xe^x$
31. $\log_{1/3} 9 = x$
32. $\log_x 8 = -3$
33. $\log_9 x = \frac{3}{2}$

Solve Problems 34–41 for x to four decimal places.

34. $x = 3(e^{1.49})$
35. $x = 230(10^{-0.161})$
36. $\log x = -2.0144$
37. $\ln x = 0.3618$
38. $35 = 7(3^x)$
39. $0.01 = e^{-0.05x}$
40. $8{,}000 = 4{,}000(1.08^x)$
41. $5^{2x-3} = 7.08$

42. Find the domain of each function:

(A) $f(x) = \frac{2x - 5}{x^2 - x - 6}$ (B) $g(x) = \frac{3x}{\sqrt{5 - x}}$

43. Find the vertex form for $f(x) = 4x^2 + 4x - 3$ and then find the intercepts, the vertex, the maximum or minimum, and the range.

44. Let $f(x) = e^x - 1$ and $g(x) = \ln(x + 2)$. Find all points of intersection for the graphs of f and g. Round answers to two decimal places.

In Problems 45 and 46, use point-by-point plotting to sketch the graph of each function.

45. $f(x) = \frac{50}{x^2 + 1}$
46. $f(x) = \frac{-66}{2 + x^2}$

If $f(x) = 5x + 1$, find and simplify each of the following in Problems 47–50.

47. $f(f(0))$
48. $f(f(-1))$
49. $f(2x - 1)$
50. $f(4 - x)$

51. Let $f(x) = 3 - 2x$. Find

(A) $f(2)$ (B) $f(2 + h)$
(C) $f(2 + h) - f(2)$ (D) $\frac{f(2 + h) - f(2)}{h}, h \neq 0$

52. Let $f(x) = x^2 - 3x + 1$. Find

(A) $f(a)$ (B) $f(a + h)$
(C) $f(a + h) - f(a)$ (D) $\frac{f(a + h) - f(a)}{h}, h \neq 0$

53. Explain how the graph of $m(x) = -|x - 4|$ is related to the graph of $y = |x|$.

54. Explain how the graph of $g(x) = 0.3x^3 + 3$ is related to the graph of $y = x^3$.

55. The following graph is the result of applying a sequence of transformations to the graph of $y = x^2$. Describe the transformations verbally and write an equation for the graph.

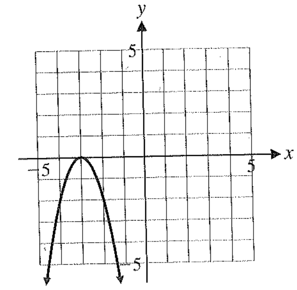


Figure for 55

56. The graph of a function f is formed by vertically stretching the graph of $y = \sqrt{x}$ by a factor of 2, and shifting it to the left 3 units and down 1 unit. Find an equation for function f and graph it for $-5 \le x \le 5$ and $-5 \le y \le 5$.

In Problems 57–59, find the equation of any horizontal asymptote.

57. $f(x) = \frac{5x + 4}{x^2 - 3x + 1}$
58. $f(x) = \frac{3x^2 + 2x - 1}{4x^2 - 5x + 3}$
59. $f(x) = \frac{x^2 + 4}{100x + 1}$

In Problems 60 and 61, find the equations of any vertical asymptotes.

60. $f(x) = \frac{x^2 + 100}{x^2 - 100}$
61. $f(x) = \frac{x^2 + 3x}{x^2 + 2x}$

In Problems 62–67, discuss the validity of each statement. If the statement is always true, explain why. If not, give a counter example.

62. Every polynomial function is a rational function.

63. Every rational function is a polynomial function.

64. The graph of every rational function has at least one vertical asymptote.

65. The graph of every exponential function has a horizontal asymptote.

66. The graph of every logarithmic function has a vertical asymptote.

67. There exists a rational function that has both a vertical and horizontal asymptote.

68. Sketch the graph of f for $x \ge 0$.

$$f(x) = \begin{cases} 9 + 0.3x & \text{if } 0 \le x \le 20 \\ 5 + 0.2x & \text{if } x > 20 \end{cases}$$

69. Sketch the graph of g for $x \ge 0$.

$$g(x) = \begin{cases} 0.5x + 5 & \text{if } 0 \le x \le 10 \\ 1.2x - 2 & \text{if } 10 < x \le 30 \\ 2x - 26 & \text{if } x > 30 \end{cases}$$

70. Write an equation for the graph shown in the form $y = a(x - h)^2 + k$, where a is either -1 or $+1$ and h and k are integers.

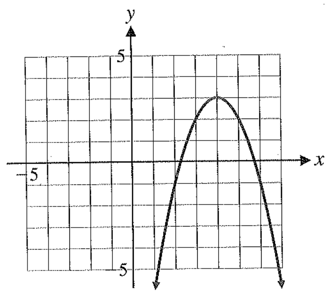


Figure for 77

71. Given $f(x) = -0.4x^2 + 3.2x + 1.2$, find the following algebraically (to three decimal places) without referring to a graph:

(A) Intercepts

(B) Vertex

(C) Maximum or minimum

(D) Range

72. Graph $f(x) = -0.4x^2 + 3.2x + 1.2$ in a graphing calculator and find the following (to three decimal places) using TRACE and appropriate commands:

(A) Intercepts

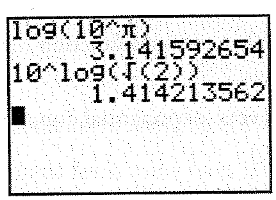
(B) Vertex

(C) Maximum or minimum

(D) Range

C

73. Noting that $\pi = 3.141\,592\,654\ldots$ and $\sqrt{2} = 1.414\,213\,562\ldots$ explain why the calculator results shown here are obvious. Discuss similar connections between the natural logarithmic function and the exponential function with base e.



Solve Problems 74–77 exactly without using a calculator.

74. $\log x - \log 3 = \log 4 - \log(x + 4)$

75. $\ln(2x - 2) - \ln(x - 1) = \ln x$

76. $\ln(x + 3) - \ln x = 2 \ln 2$

77. $\log 3x^2 = 2 + \log 9x$

78. Write $\ln y = -5t + \ln c$ in an exponential form free of logarithms. Then solve for y in terms of the remaining variables.

79. Explain why 1 cannot be used as a logarithmic base.

80. The following graph is the result of applying a sequence of transformations to the graph of $y = \sqrt[3]{x}$. Describe the transformations verbally, and write an equation for the graph.

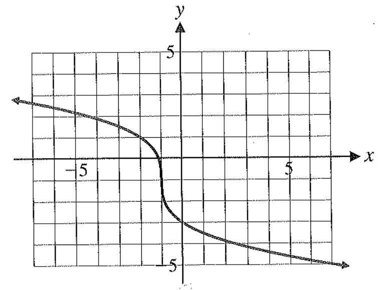


Figure for 80

81. Given $G(x) = 0.3x^2 + 1.2x - 6.9$, find the following algebraically (to three decimal places) without the use of a graph:

(A) Intercepts

(B) Vertex

(C) Maximum or minimum

(D) Range

82. Graph $G(x) = 0.3x^2 + 1.2x - 6.9$ in a standard viewing window. Then find each of the following (to three decimal places) using appropriate commands.

(A) Intercepts

(B) Vertex

(C) Maximum or minimum

(D) Range

Applications

In all problems involving days, a 365-day year is assumed.

83. Electricity rates. The table shows the electricity rates charged by Easton Utilities in the summer months.

(A) Write a piecewise definition of the monthly charge $S(x)$ (in dollars) for a customer who uses x kWh in a summer month.

(B) Graph $S(x)$.

Energy Charge (June–September)
\$3.00 for the first 20 kWh or less
5.70¢ per kWh for the next 180 kWh
3.46¢ per kWh for the next 800 kWh
2.17¢ per kWh for all over 1,000 kWh

84. Money growth. Provident Bank of Cincinnati, Ohio recently offered a certificate of deposit that paid 5.35% compounded quarterly. If a \$5,000 CD earns this rate for 5 years, how much will it be worth?

85. Money growth. Capital One Bank of Glen Allen, Virginia recently offered a certificate of deposit that paid 4.82% compounded daily. If a \$5,000 CD earns this rate for 5 years, how much will it be worth?

86. Money growth. How long will it take for money invested at 6.59% compounded monthly to triple?

87. Money growth. How long will it take for money invested at 7.39% compounded continuously to double?

88. Break-even analysis. The research department in a company that manufactures AM/FM clock radios established the following price–demand, cost, and revenue functions:

$p(x) = 50 - 1.25x$	Price–demand function
$C(x) = 160 + 10x$	Cost function
$R(x) = xp(x)$	
$= x(50 - 1.25x)$	Revenue function

where x is in thousands of units, and $C(x)$ and $R(x)$ are in thousands of dollars. All three functions have domain $1 \le x \le 40$.

(A) Graph the cost function and the revenue function simutaneously in the same coordinate system.

(B) Determine algebraically when $R = C$. Then, with the aid of part (A), determine when $R < C$ and $R > C$ to the nearest unit.

(C) Determine algebraically the maximum revenue (to the nearest thousand dollars) and the output (to the nearest unit) that produces the maximum revenue. What is the wholesale price of the radio (to the nearest dollar) at this output?

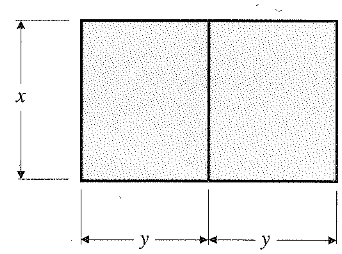
89. Profit–loss analysis. Use the cost and revenue functions from Problem 88.

(A) Write a profit function and graph it in a graphing calculator.

(B) Determine graphically when $P = 0, P < 0$, and $P > 0$ to the nearest unit.

(C) Determine graphically the maximum profit (to the nearest thousand dollars) and the output (to the nearest unit) that produces the maximum profit. What is the wholesale price of the radio (to the nearest dollar) at this output? [Compare with Problem 88C.]

90. Construction. A construction company has 840 feet of chain-link fence that is used to enclose storage areas for equipment and materials at construction sites. The supervisor wants to set up two identical rectangular storage areas sharing a common fence (see the figure).



Assuming that all fencing is used,

(A) Express the total area $A(x)$ enclosed by both pens as a function of x.

(B) From physical considerations, what is the domain of the function A?

(C) Graph function A in a rectangular coordinate system.

(D) Use the graph to discuss the number and approximate locations of values of x that would produce storage areas with a combined area of 25,000 square feet.

(E) Approximate graphically (to the nearest foot) the values of x that would produce storage areas with a combined area of 25,000 square feet.

(F) Determine algebraically the dimensions of the storage areas that have the maximum total combined area. What is the maximum area?

91. Equilibrium point. A company is planning to introduce a 10-piece set of nonstick cookware. A marketing company established price–demand and price–supply tables for selected prices (Tables 1 and 2), where x is the number of cookware sets people are willing to buy and the company is willing to sell each month at a price of p dollars per set.

Table 1 Price–Demand

x	$p = D(x)$ (\$)
985	330
2,145	225
2,950	170
4,225	105
5,100	50

Table 2 Price–Supply

x	$p = S(x)$ (\$)
985	30
2,145	75
2,950	110
4,225	155
5,100	190

(A) Find a quadratic regression model for the data in Table 1. Estimate the demand at a price level of \$180.

(B) Find a linear regression model for the data in Table 2. Estimate the supply at a price level of \$180.

(C) Does a price level of \$180 represent a stable condition, or is the price likely to increase or decrease? Explain.

(D) Use the models in parts (A) and (B) to find the equilibrium point. Write the equilibrium price to the nearest cent and the equilibrium quantity to the nearest unit.

92. Crime statistics. According to data published by the FBI, the crime index in the United States has shown a downward trend since the early 1990s (Table 3).

Table 3 Crime Index

Year	Crimes per 100,000 Inhabitants
1987	5,550
1992	5,660
1997	4,930
2002	4,119
2007	3,016

(A) Find a cubic regression model for the crime index if $x = 0$ represents 1987.

(B) Use the cubic regression model to predict the crime index in 2017.

93. Medicine. One leukemic cell injected into a healthy mouse will divide into 2 cells in about $\frac{1}{2}$ day. At the end of the day these 2 cells will divide into 4. This doubling continues until 1 billion cells are formed; then the animal dies with leukemic cells in every part of the body.

(A) Write an equation that will give the number N of leukemic cells at the end of t days.

(B) When, to the nearest day, will the mouse die?

94. Marine biology. The intensity of light entering water is reduced according to the exponential equation

$$I = I_0 e^{-kd}$$

where I is the intensity d feet below the surface, I_0 is the intensity at the surface, and k is the coefficient of extinction. Measurements in the Sargasso Sea have indicated that half of the surface light reaches a depth of 73.6 feet. Find k (to five decimal places), and find the depth (to the nearest foot) at which 1% of the surface light remains.

95. Agriculture. The number of dairy cows on farms in the United States is shown in Table 4 for selected years since 1950. Let 1940 be year 0.

Table 4 Dairy Cows on Farms in the United States

Year	Dairy Cows (thousands)
1950	23,853
1960	19,527
1970	12,091
1980	10,758
1990	10,015
2000	9,190

(A) Find a logarithmic regression model ($y = a + b \ln x$) for the data. Estimate (to the nearest thousand) the number of dairy cows in 2020.

(B) Explain why it is not a good idea to let 1950 be year 0.

96. Population growth. The population of some countries has a relative growth rate of 3% (or more) per year. At this rate, how many years (to the nearest tenth of a year) will it take a population to double?

97. Medicare. The annual expenditures for Medicare (in billions of dollars) by the U.S. government for selected years since 1980 are shown in Table 5. Let x represent years since 1980.

Table 5 Medicare Expenditures

Year	Billion $
1980	37
1985	72
1990	111
1995	181
2000	197
2005	330

(A) Find an exponential regression model ($y = ab^x$) for the data. Estimate (to the nearest billion) the annual expenditures in 2017.

(B) When will the annual expenditures reach one trillion dollars?