

1. Let $f(x) = \frac{x^2 + 1}{x^2 - 1}$. Find each of the following:

a) $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} \frac{x^2 + 1}{x^2 - 1} = \frac{1}{1} = \boxed{1}$

b) $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} \frac{x^2 + 1}{(x+1)(x-1)} = \frac{1^2 + 1}{(1+1)(1^- - 1)} = \frac{2}{2 \cdot 0^-} = \boxed{-\infty}$

c) $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \frac{x^2 + 1}{(x+1)(x-1)} = \frac{1^2 + 1}{(1+1)(1^+ - 1)} = \frac{2}{2 \cdot 0^+} = \boxed{+\infty}$

d) $\lim_{x \rightarrow 1} f(x) = \text{DNE}$

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2. Use the *limit definition* of derivative to find $f'(x)$ for $f(x) = 2x^2 + 8$.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

1. $f(x+h) = 2(x+h)^2 + 8 = 2(x^2 + 2xh + h^2) + 8 = 2x^2 + 4xh + 2h^2 + 8$

2. $f(x+h) - f(x) = 2x^2 + 4xh + 2h^2 + 8 - (2x^2 + 8) = 4xh + 2h^2$

3. $\frac{f(x+h) - f(x)}{h} = \frac{4xh + 2h^2}{h} = \frac{h(4x + 2h)}{h} = 4x + 2h$

4. $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} (4x + 2h) = 4x + 2(0) = \boxed{4x = f'(x)}$