§5.2. Second Derivatives

Def'n. Concavity

The graph of a function f is concave up on an open interval (a,b) if f' is increasing on (a,b). Similarly, f is concave down on (a,b) if f' is decreasing on (a,b).

Conc up:

Conc down:

Second derivative:

For a function $y=f(x)$, if the second derivative of f exists, it is given by

$$f''(x) = \frac{d}{dx}\left[f'(x)\right] = \frac{d^2y}{dx^2} = y''.$$

Based on previous considerations,

f is concave up on (a,b) iff $f''>0$ on (a,b) and

f is conc down iff $f''<0$.

If $f''(c)=0$, then c is called an inflection point of f.

Ex. Intervals of concavity:

$f(x) = x^2$

$g(x) = \sqrt{x}$

$h(x) = x^3 - 6x^2 + 9x + 1$

§5.4 also) Ex. Sketch the graph: #51

$$f(x) = (x^2+3)(9-x^2)$$

1. domain, intercepts, asymptotes
2. analyze f'
3. analyze f''
4. sketch graph of f.

Ex. $g(x) = \dfrac{2x-1}{x^2}$

Ex. # 73, p. 321 $\quad$ p = price, x = units sold / wk. (demand)

$$p = 1296 - 0.12x^2 \quad \$/\text{unit} \qquad 0 \le x \le 80$$

Weekly Revenue is then: $R(x) = xp = 1296x - 0.12x^3 \quad 0 \le x \le 80$

Sketch the graph of R.